

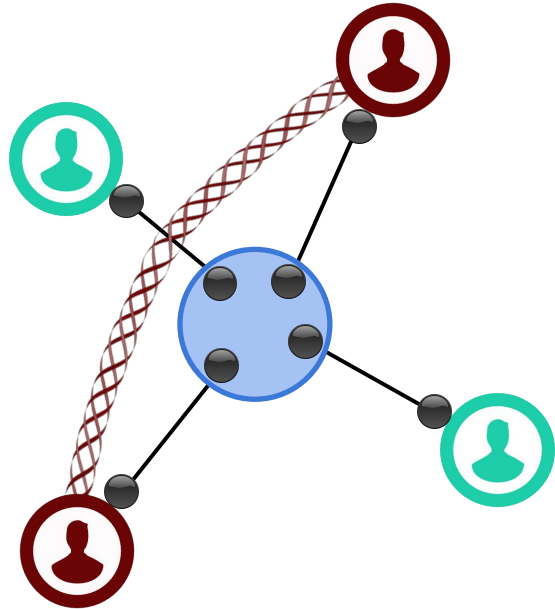
# On the Capacity Region of Bipartite and Tripartite Entanglement Switching

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Philippe Nain<sup>‡</sup>, and Don Towsley<sup>\*</sup>

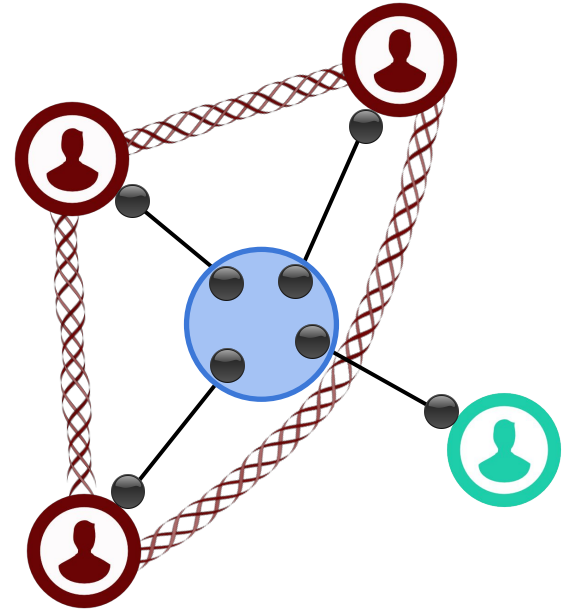
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Performance 2020

## Example of bipartite entanglement (Bell pair)



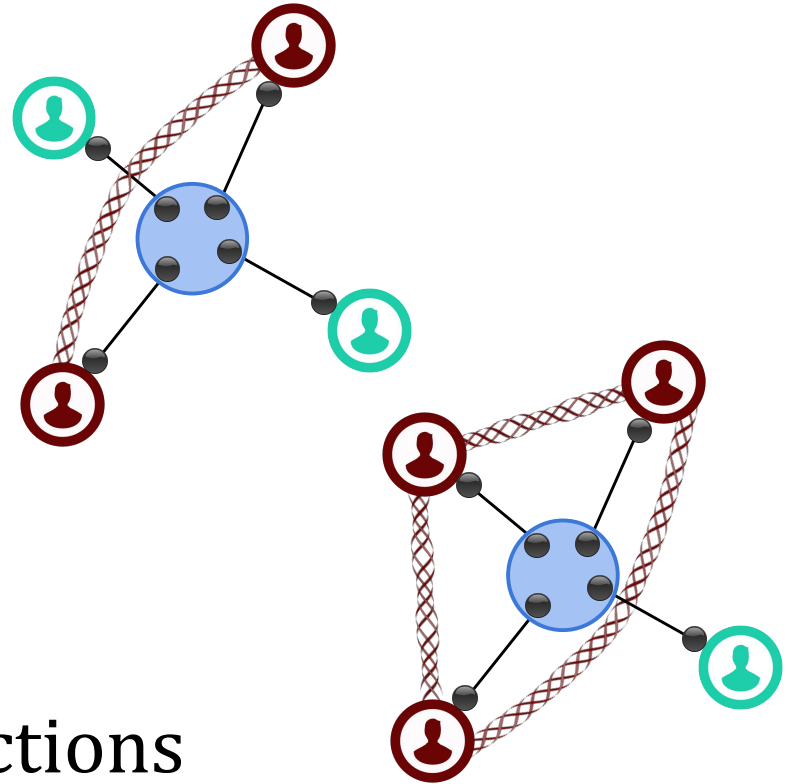
## Example of tripartite entanglement (3-qubit GHZ)



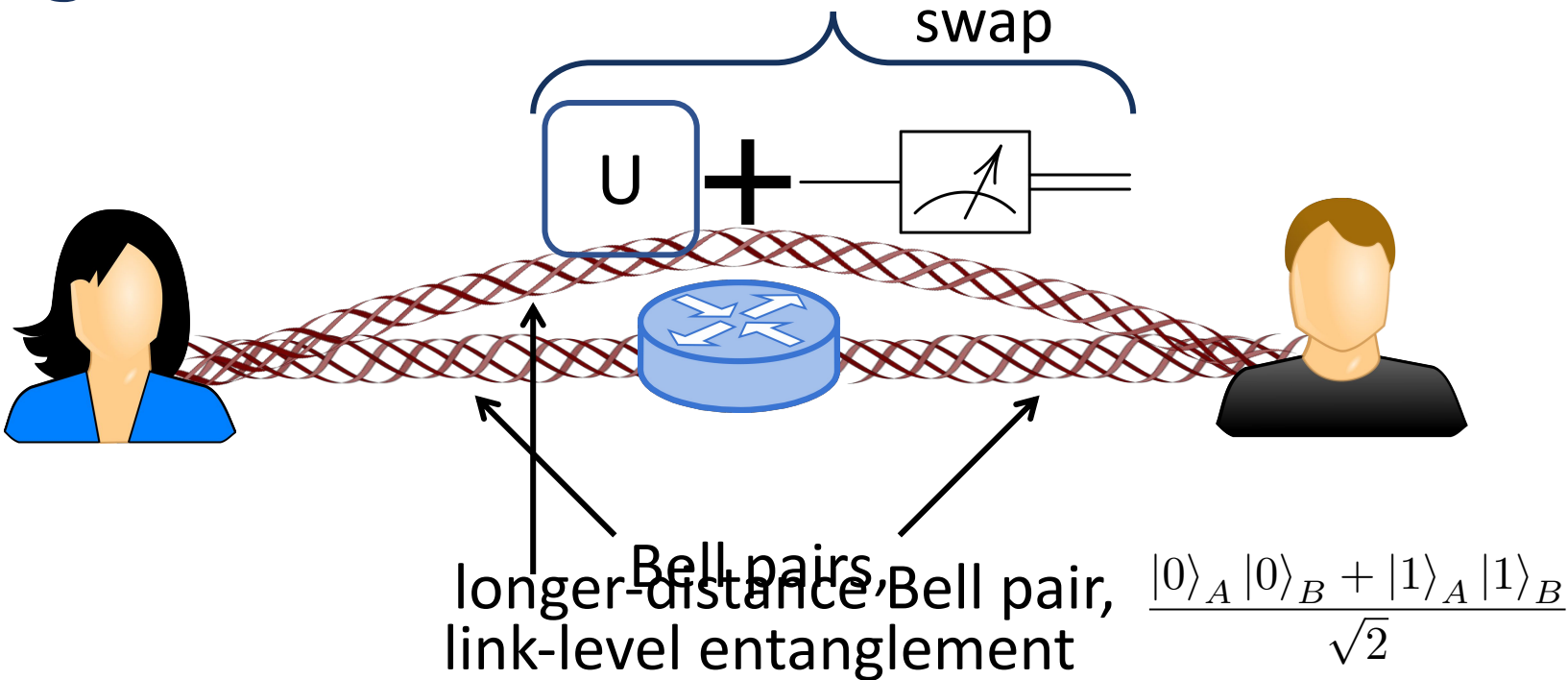
\*See first few slides of Performance 2020 talk “On the Exact Analysis of a Quantum Switch” for brief background on quantum communication and quantum entanglement switching

# Talk Outline

- Problem statement
- Switch description
- Analysis
- Results
- Summary & Future Directions



# Entanglement Distribution



- Teleportation (consumes one Bell pair)
- Use directly, *e.g.*, in E91 or similar protocols

# GHZ States

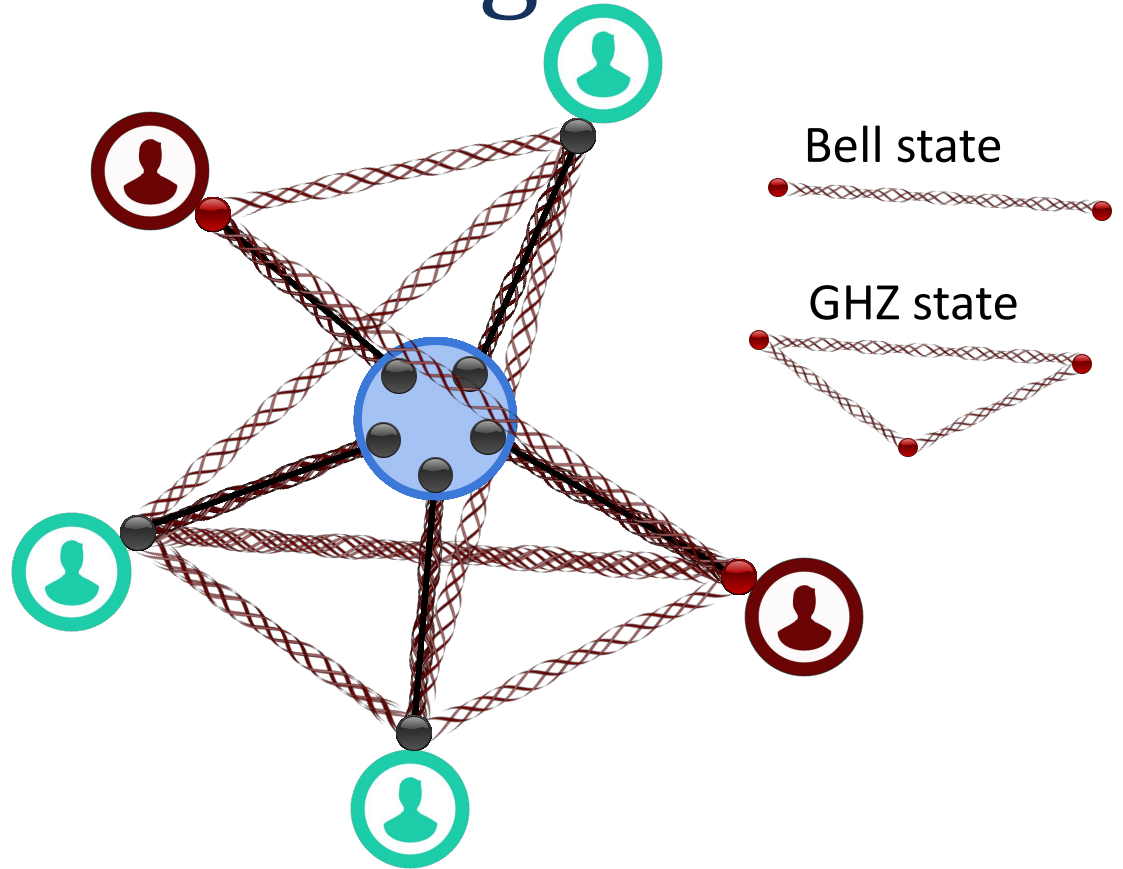


$n$ -partite GHZ state  $|GHZ\rangle = \frac{|0\rangle^{\otimes n} + |1\rangle^{\otimes n}}{\sqrt{2}}$

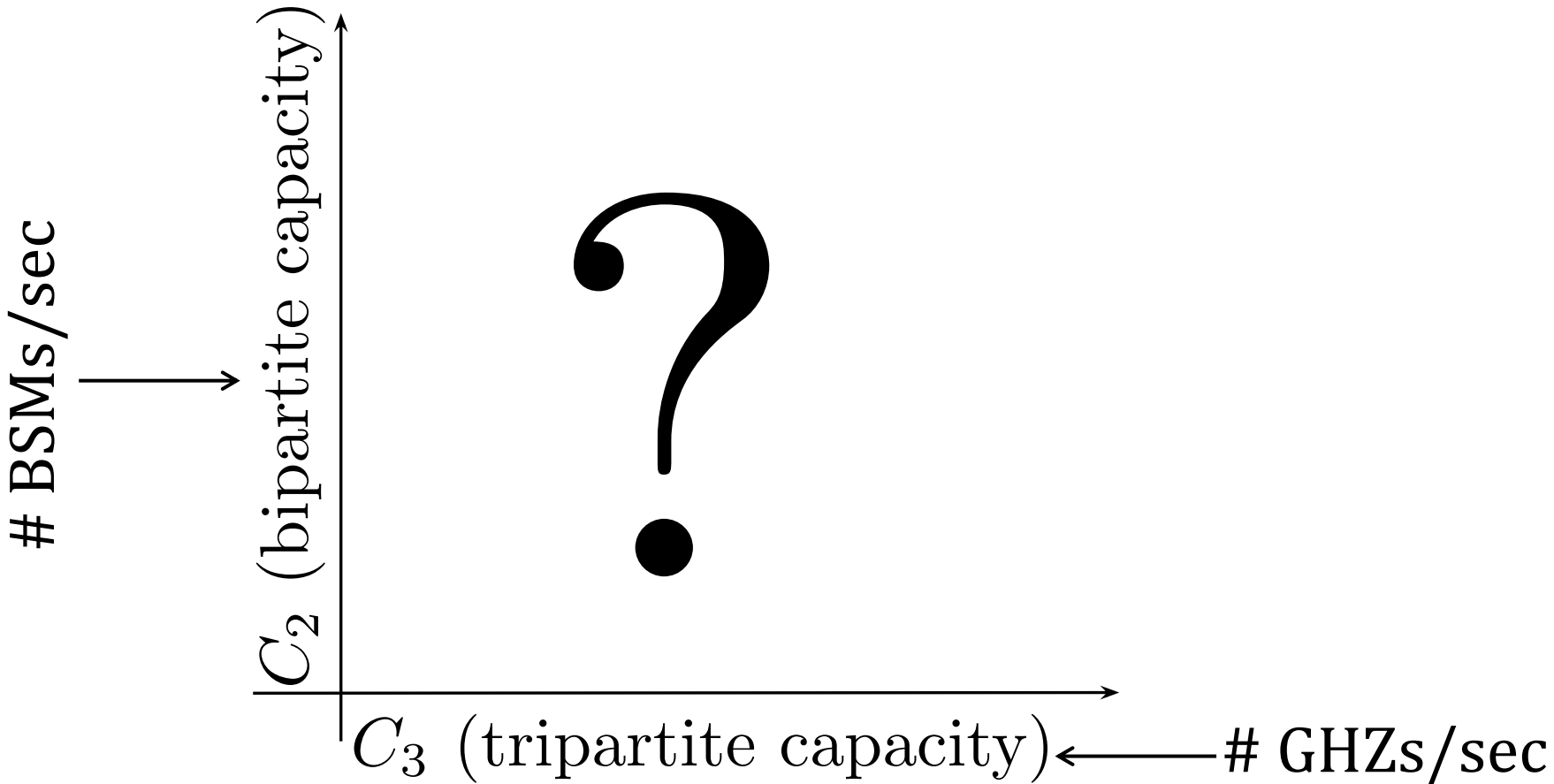
Used in multiparty QKD, secret sharing, quantum sensing, multipartite generalization of superdense coding...

# Bi-, Tripartite Switching

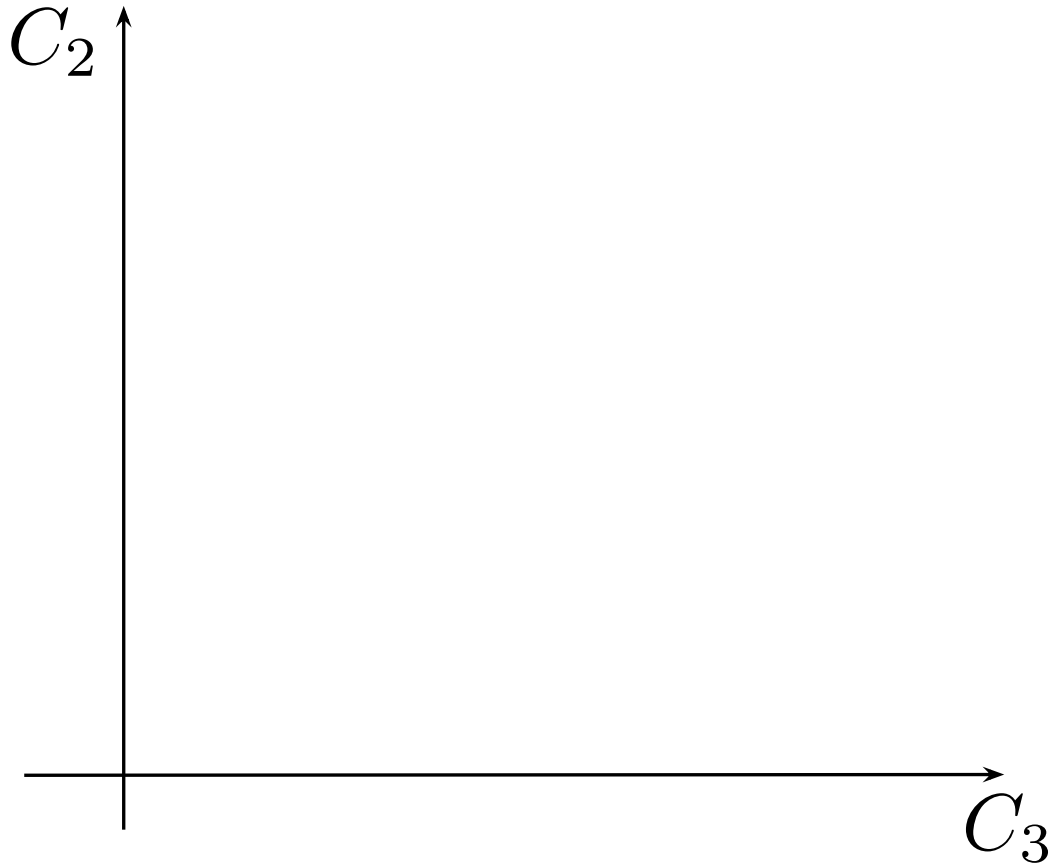
$k=5$  links  
 $n=3$  or  $2$



Q1: What is the capacity region of the switch?

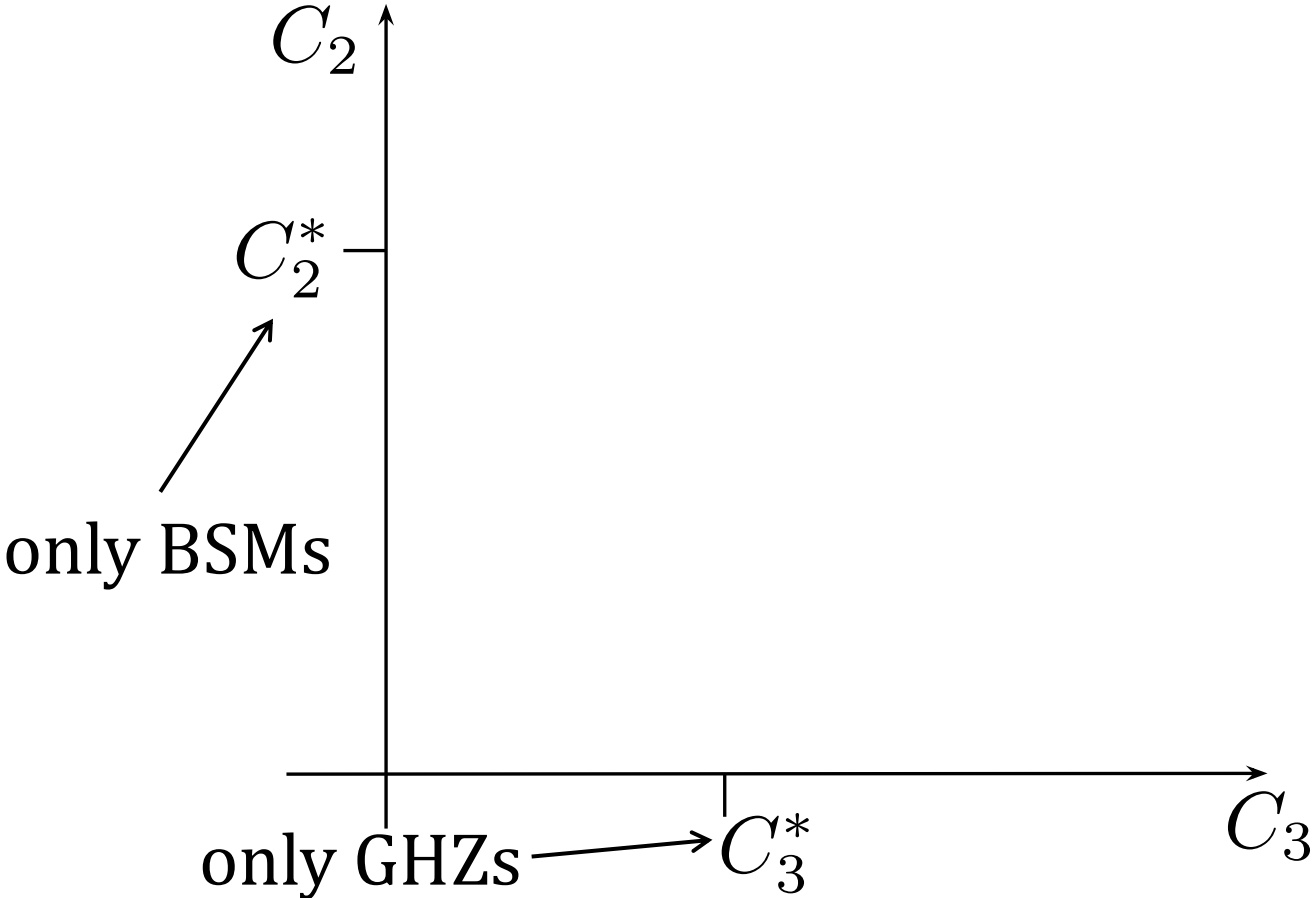


Q1: What is the capacity region of the switch?

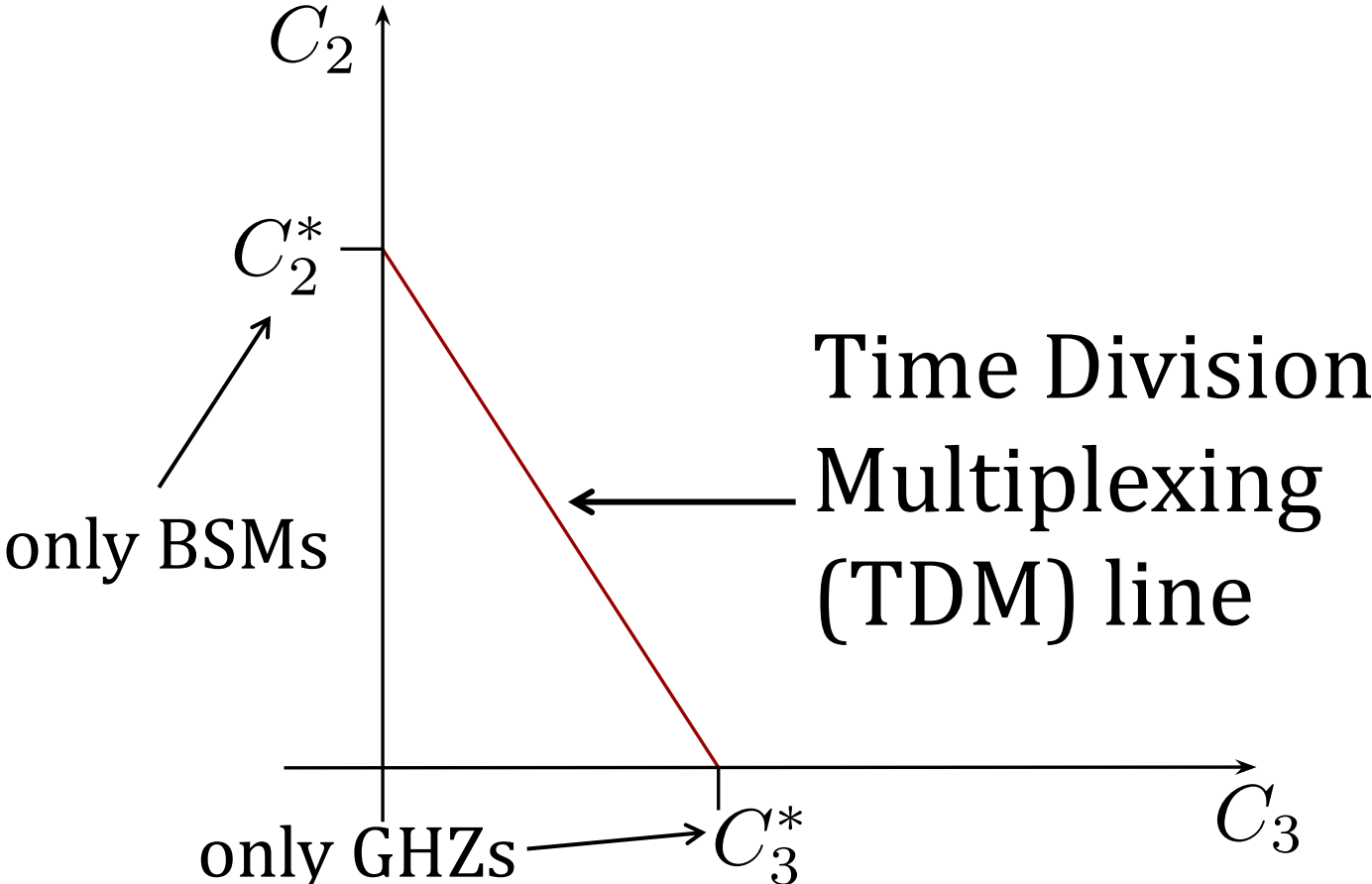




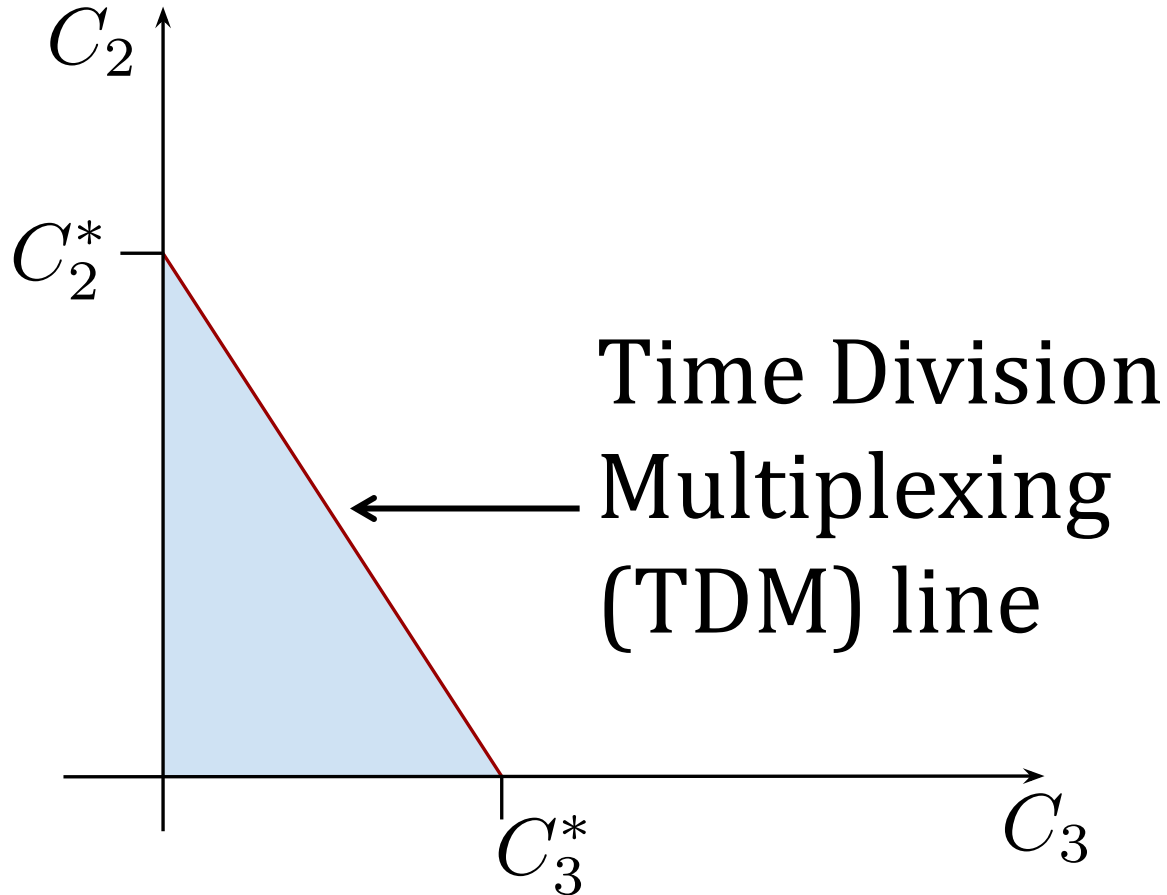
Q1: What is the capacity region of the switch?



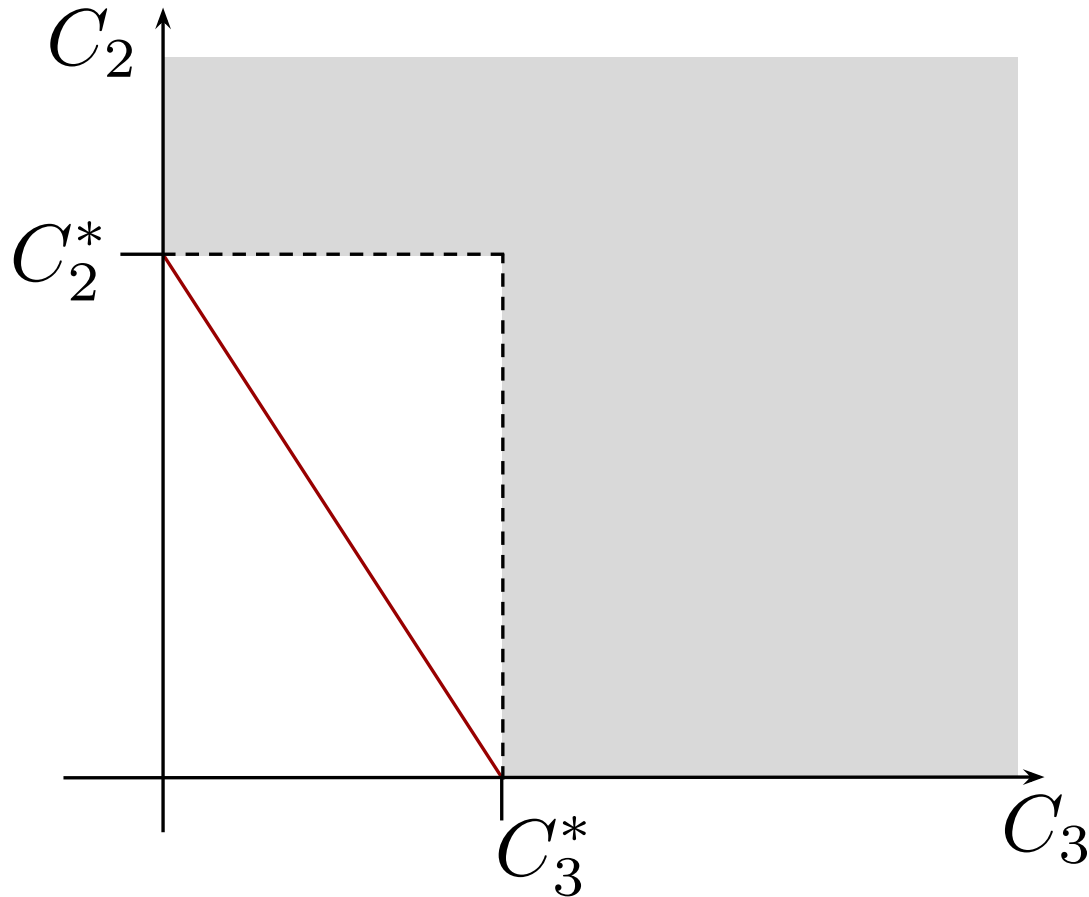
Q1: What is the capacity region of the switch?



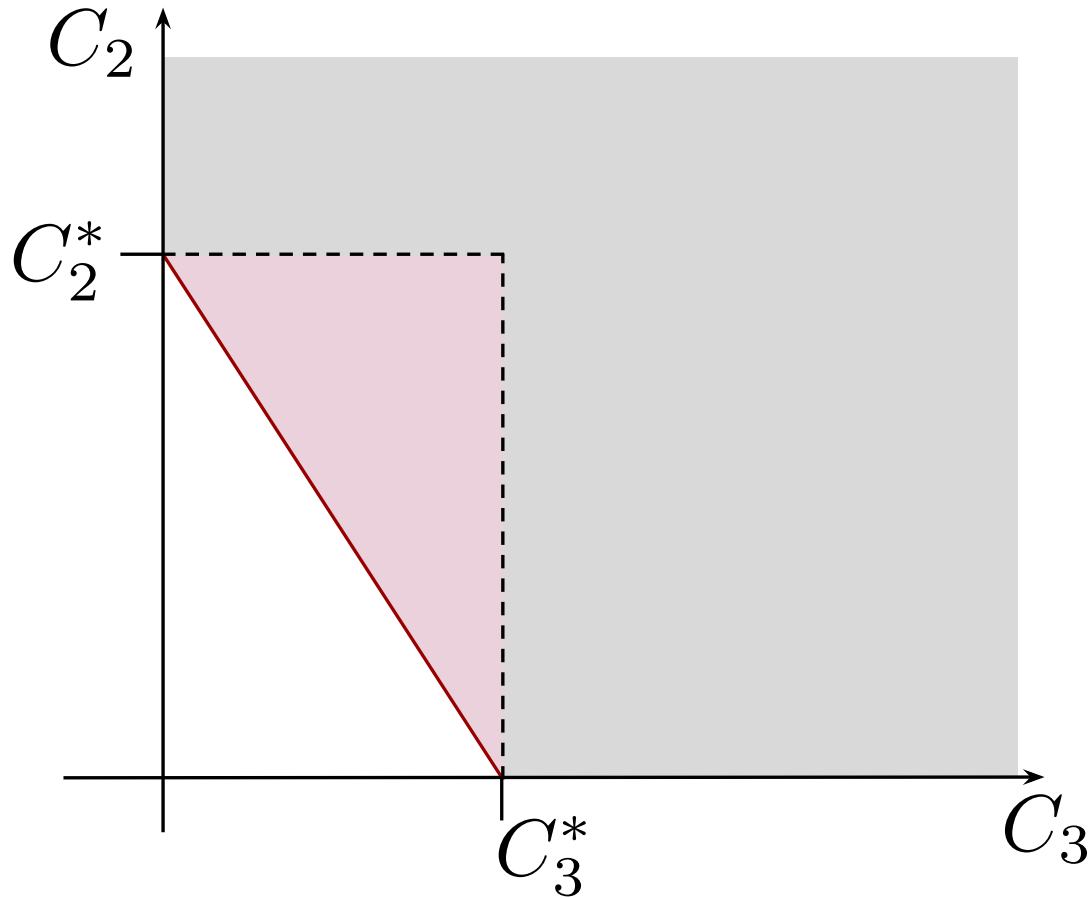
Q1: What is the capacity region of the switch?



Q2: Are there switching policies better than TDM?



Q2: Are there switching policies better than TDM?




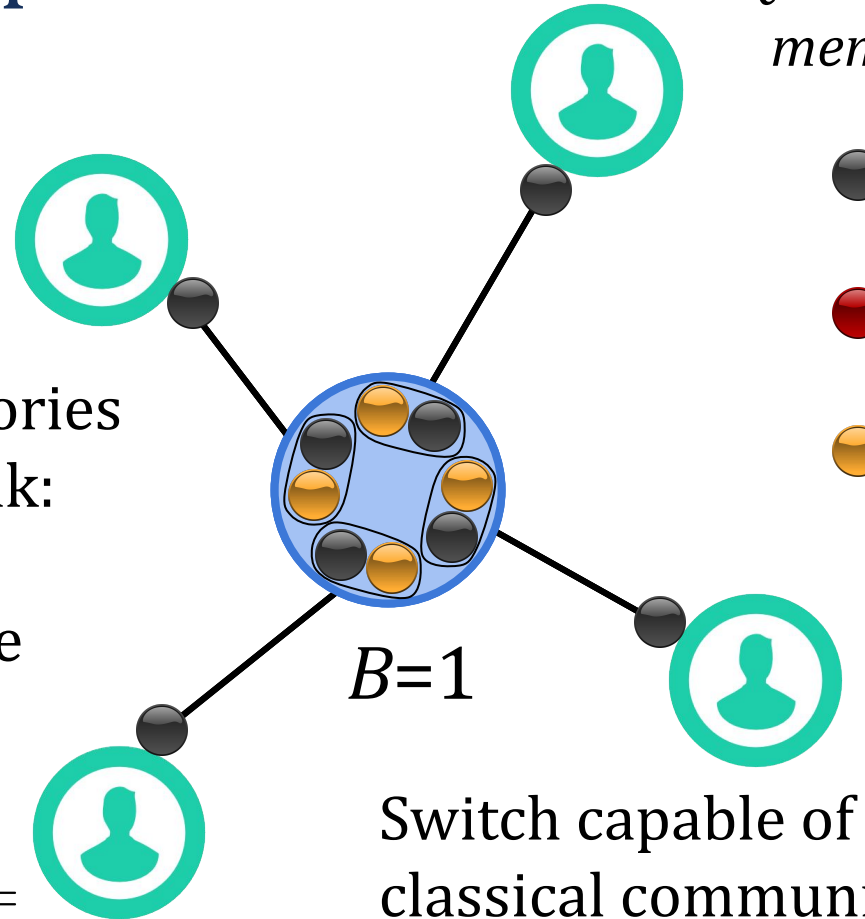
# Switch Description

Switch is equipped with quantum memories (buffer)

$B+1$  quantum memories allocated to each link:

- $B$  long-coherence
- 1 short-coherence

Switch can perform projective 



*Quantum memory*

*Type*



unoccupied, long-term



occupied, long-term



short-term

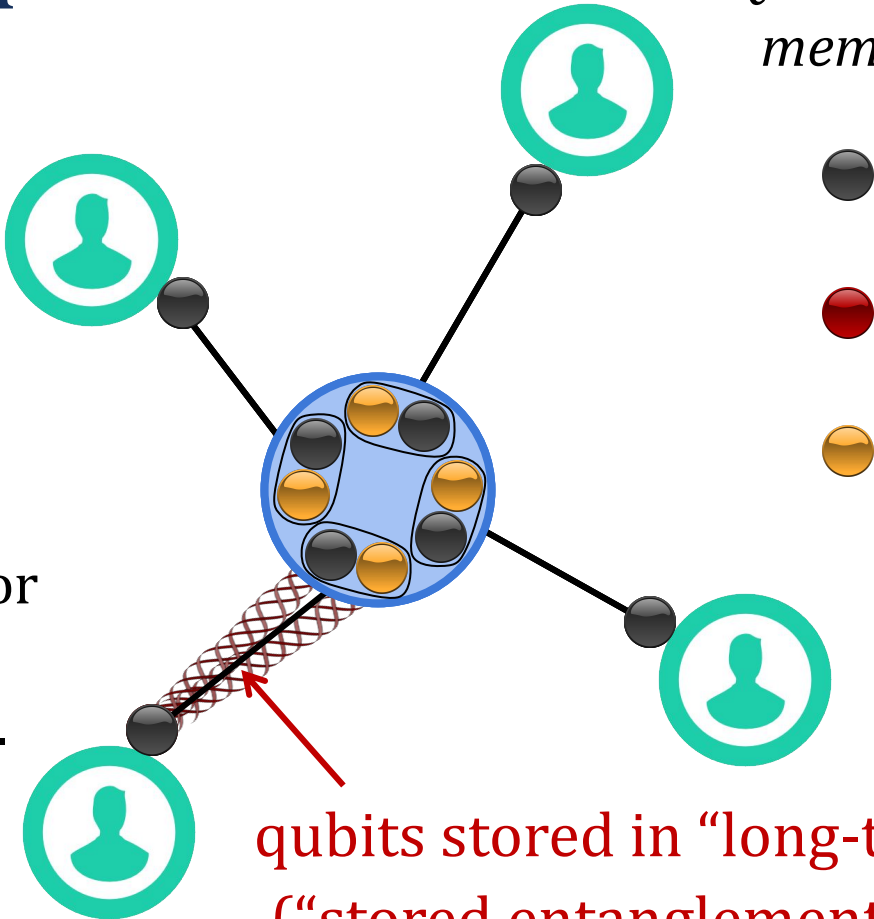
$B=1$

Switch capable of classical communication

# Switch Description

Qubits from newly-generated Bell pairs are initially stored in “short-term” memories.

Must either use qubit or store in long-term memory; else, it is lost.



*Quantum memory*

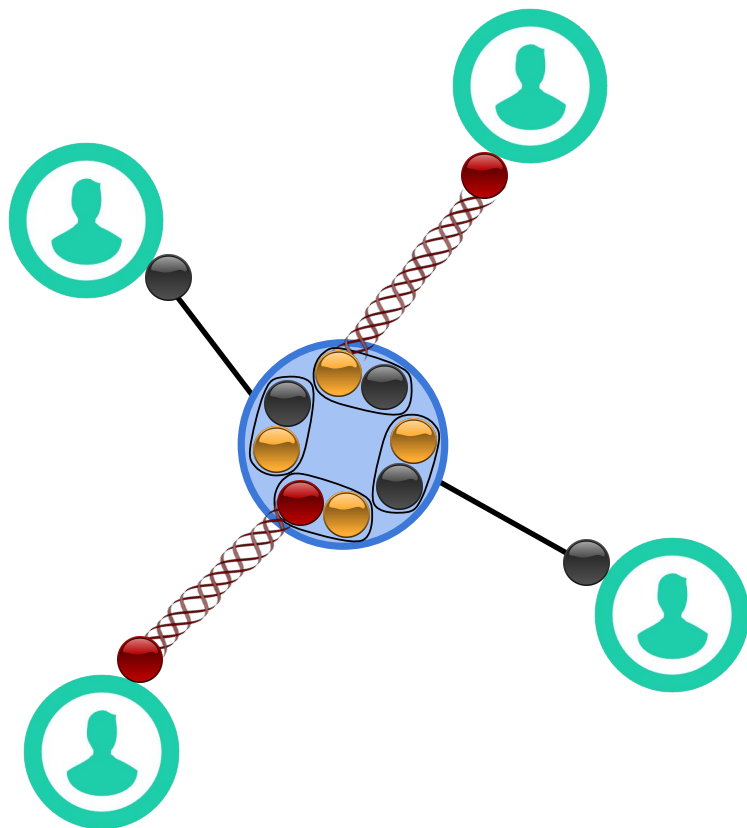
- 
- 
- 

*Type*

- unoccupied, long-term
- occupied, long-term
- short-term

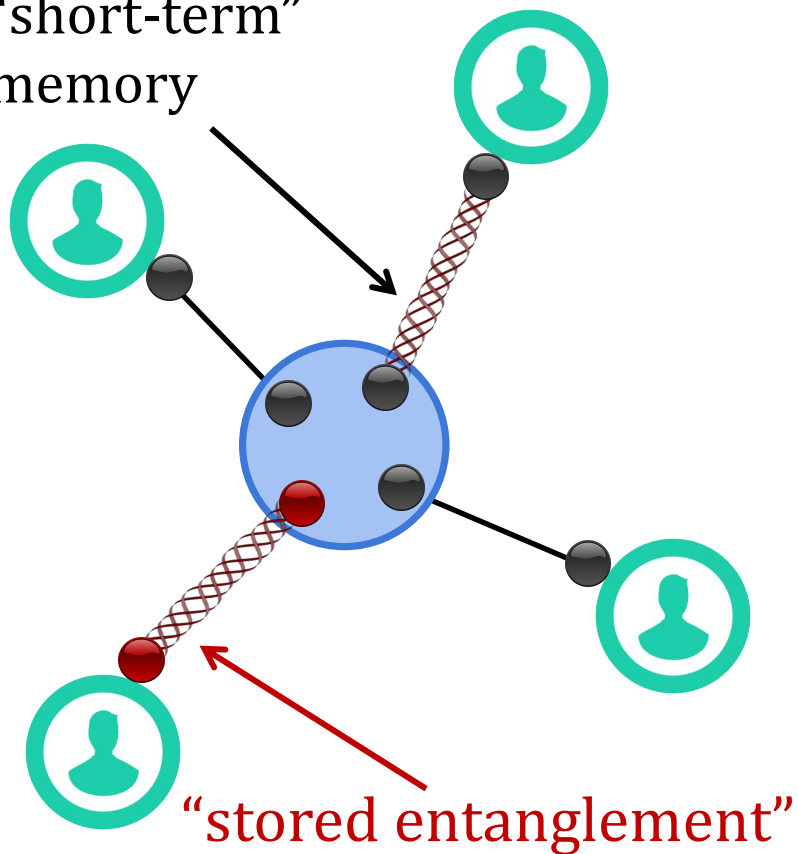
qubits stored in “long-term” memory (“stored entanglement”)

# Switch Description



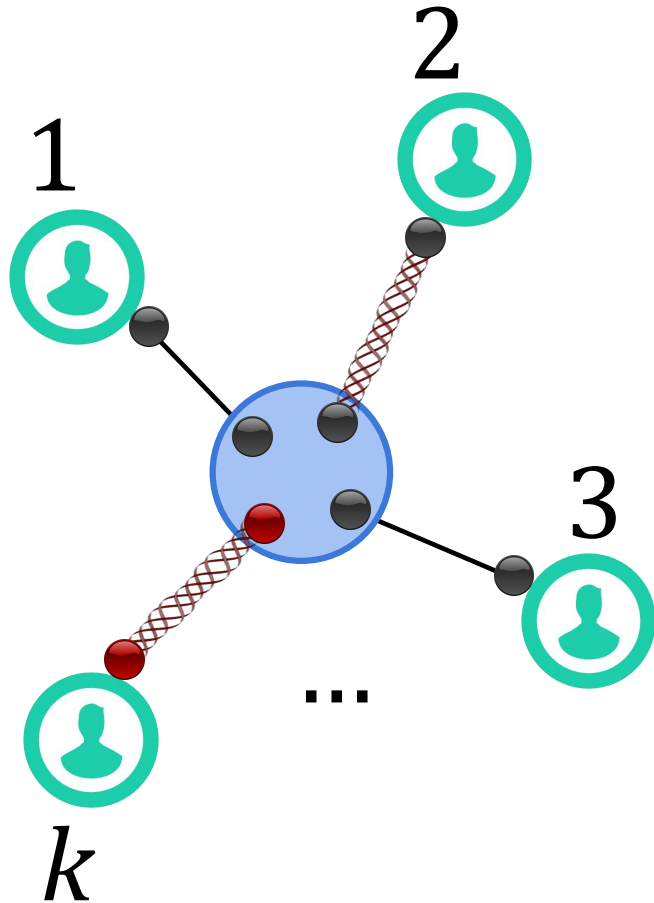
≡

qubits stored in  
“short-term”  
memory





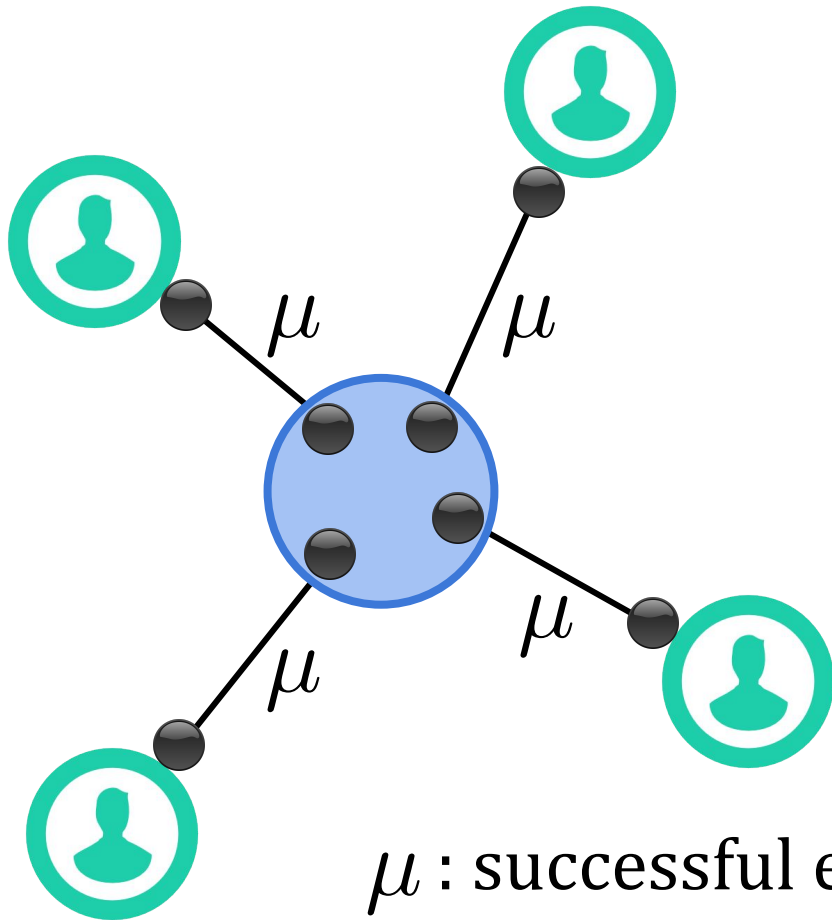
# Switch Operation



Switch serves entangled states to  $k$  users

1. Bell pairs over individual links
2. switch performs either a BSM or a GHZ measurement, resulting in end-to-end entanglement

# Modeling Link-Level Entanglement Generation

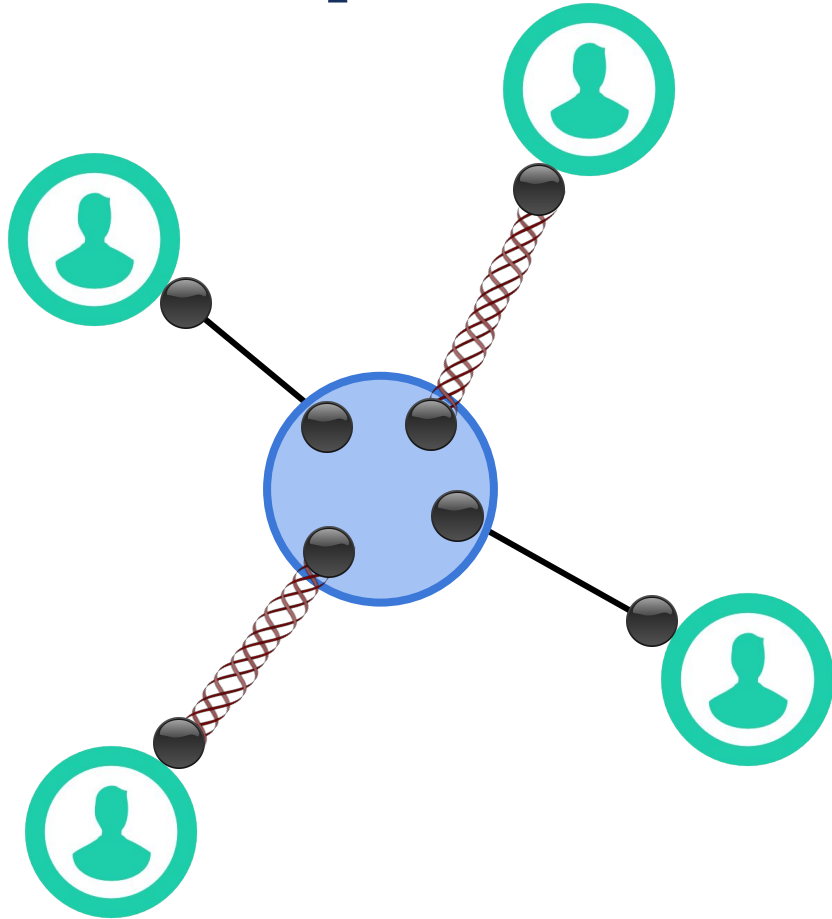


1. link-level entanglement generation

entangled states  
generated according  
to Poisson process

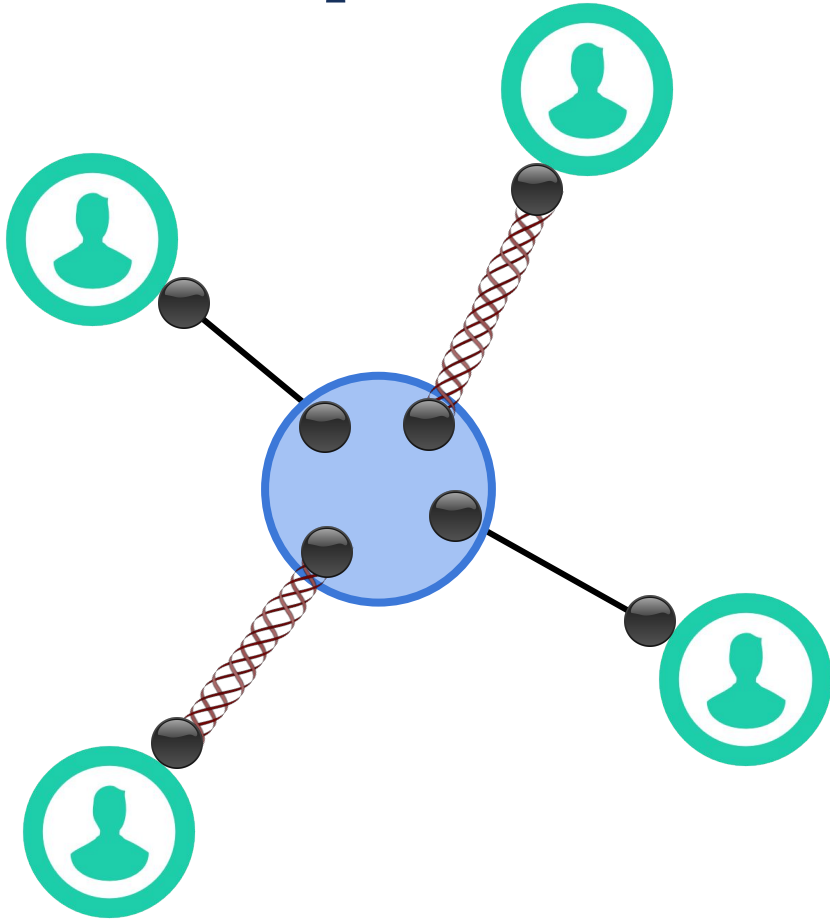
$\mu$  : successful entanglement generation rate


# Switch Operation



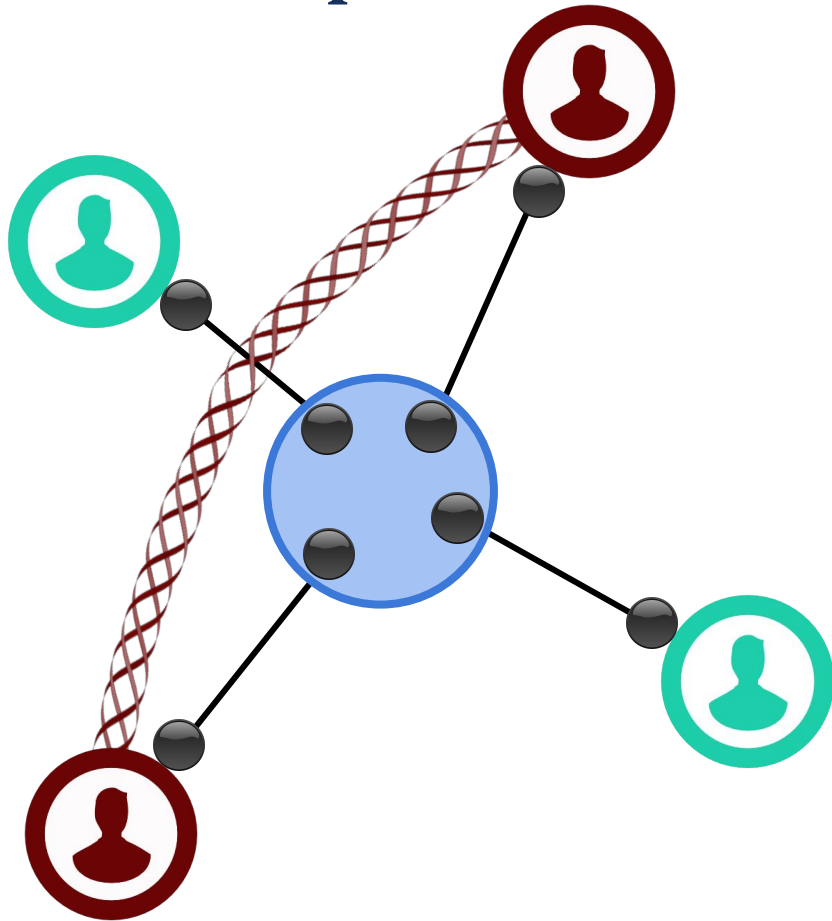
1. link-level entanglement generation
2. swapping, according to switching policy


# Switch Operation



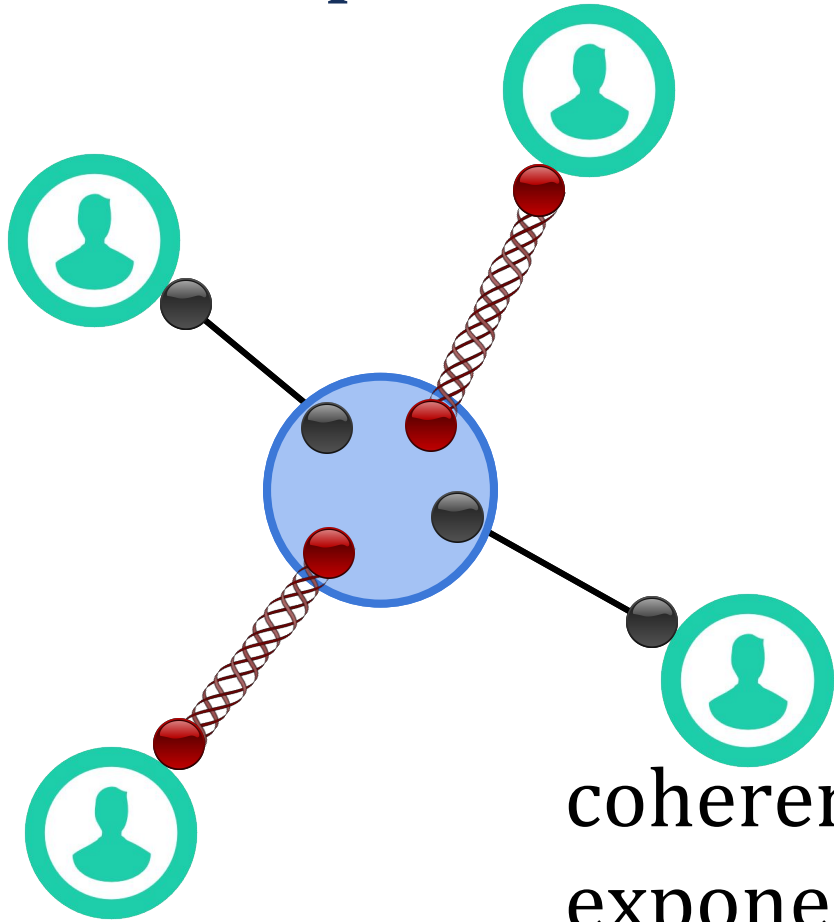
- perform a 
- store in memory
- discard


# Switch Operation



- perform a 
- store in memory
- discard

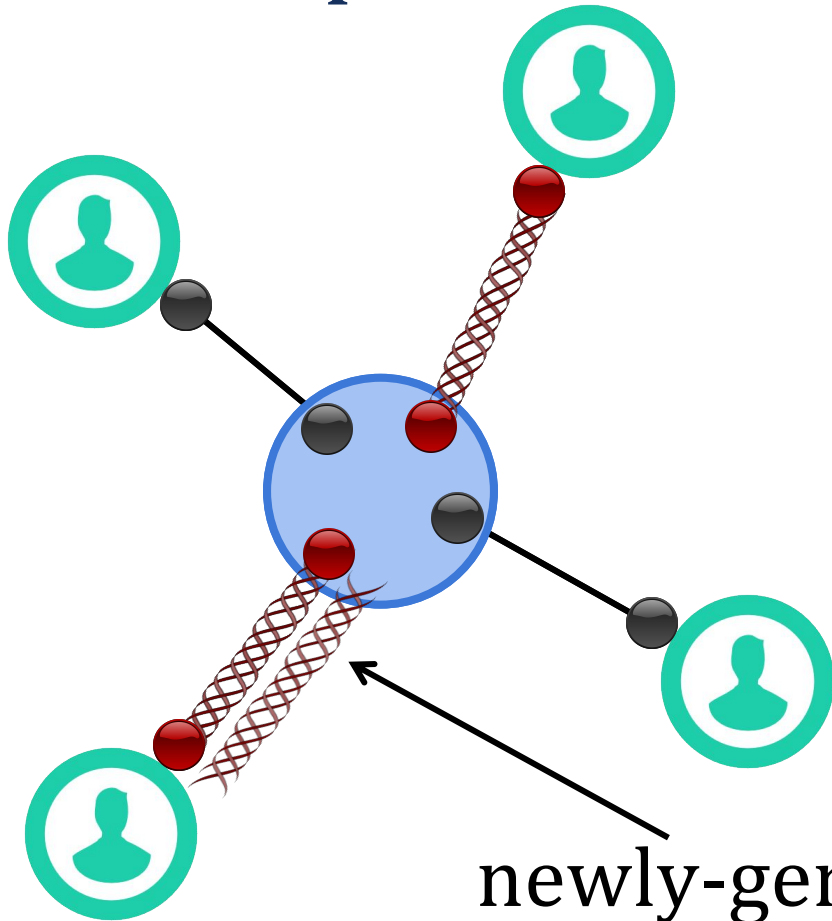
# Switch Operation

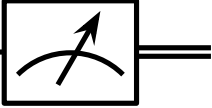


- perform a 
- store in memory
- discard

coherence time is an  
exponential r.v. with mean  $1/\alpha$

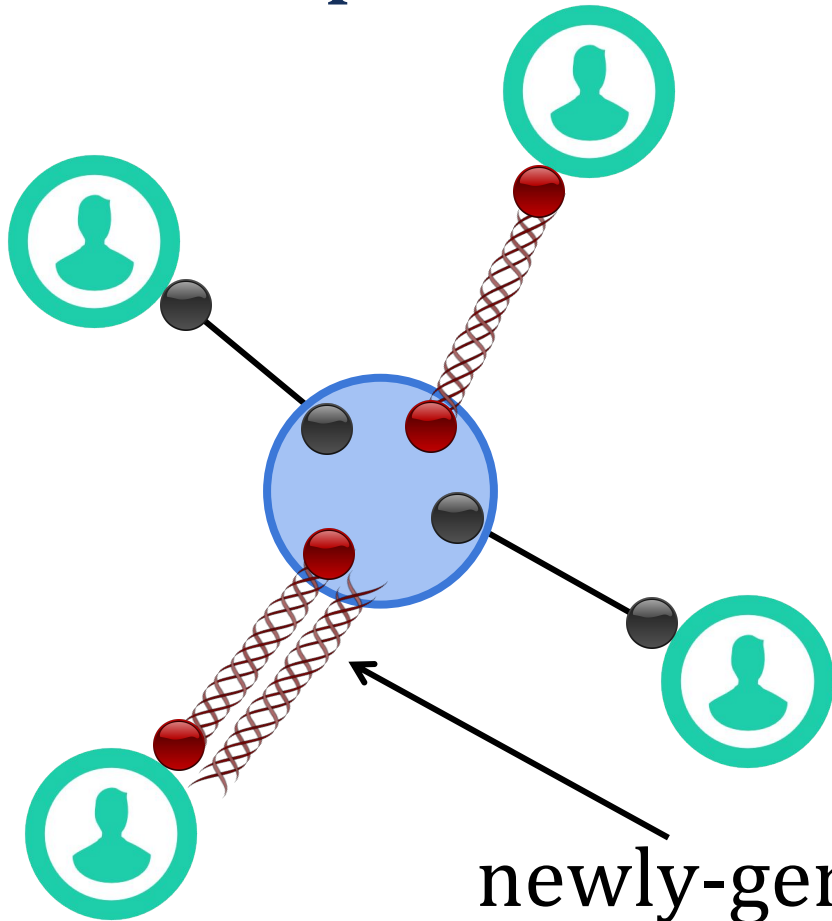
# Switch Operation




- perform a 
- store in memory
- discard

newly-generated entanglement

# Switch Operation

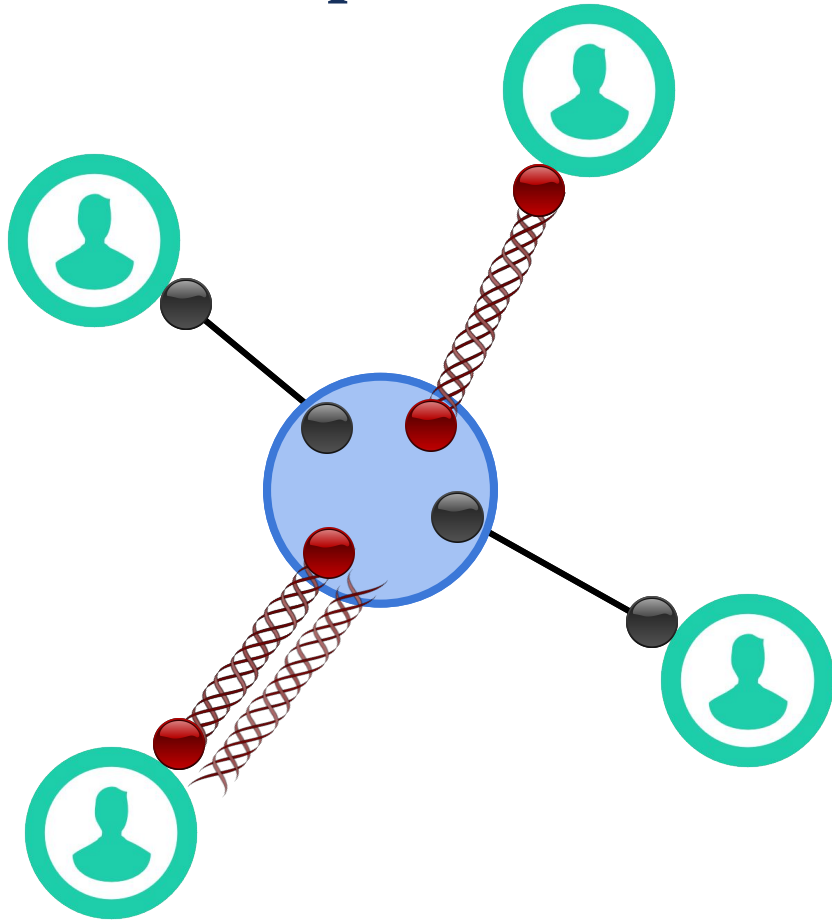



- perform a 
- store in memory
- discard

newly-generated entanglement

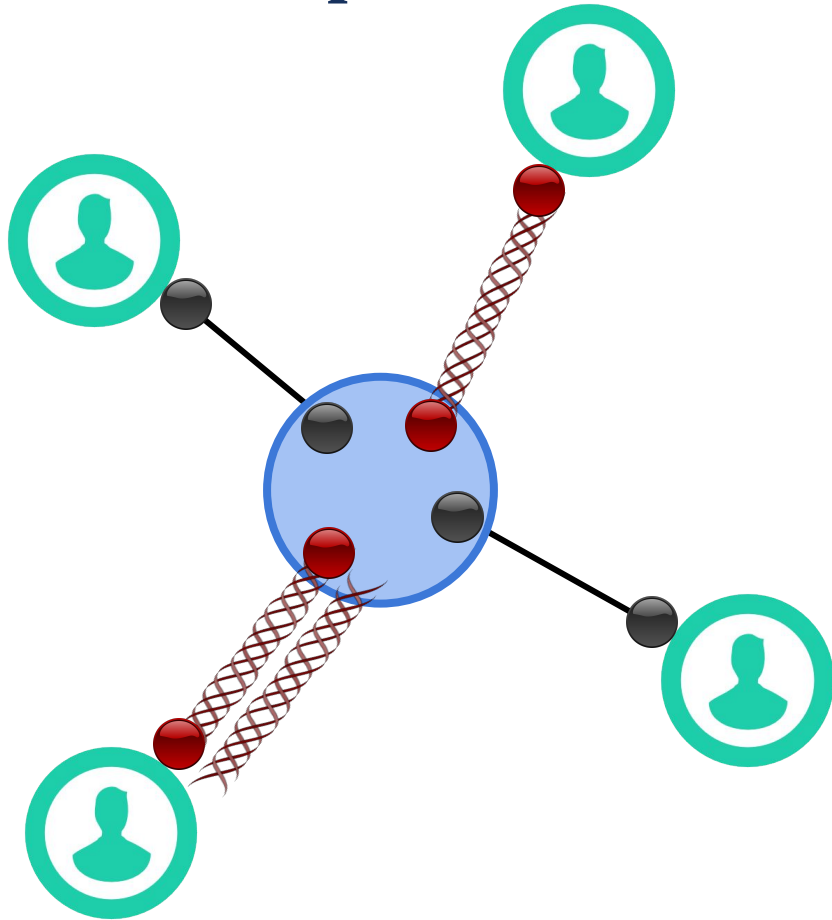


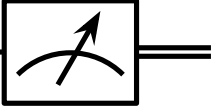
# Switch Operation



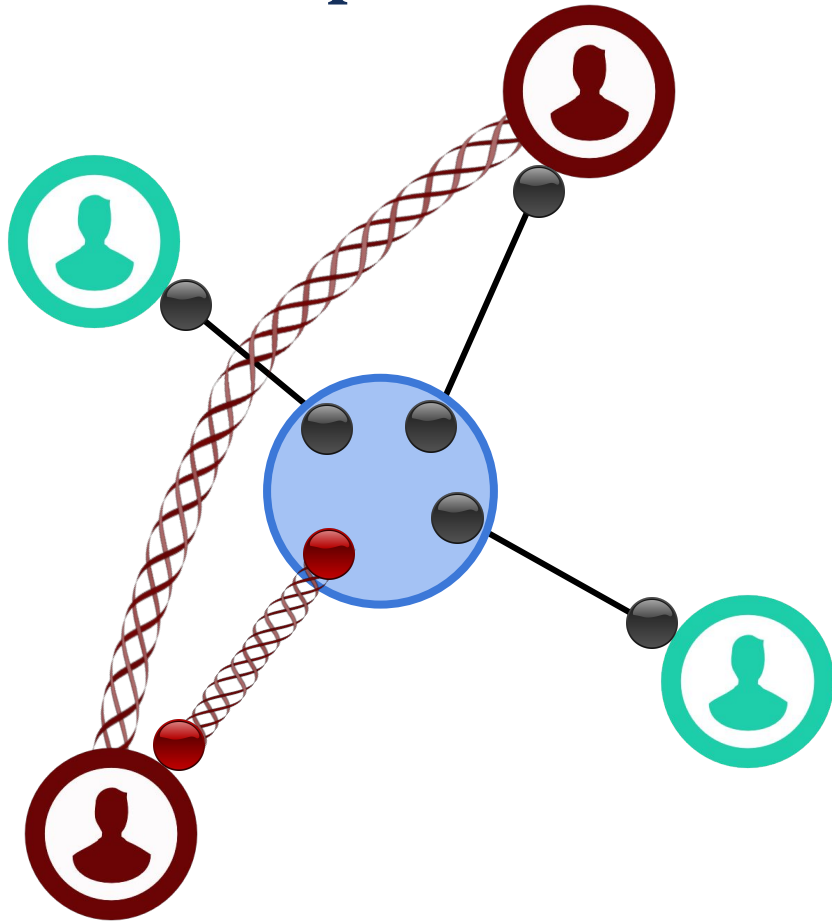
- perform a 
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# Switch Operation



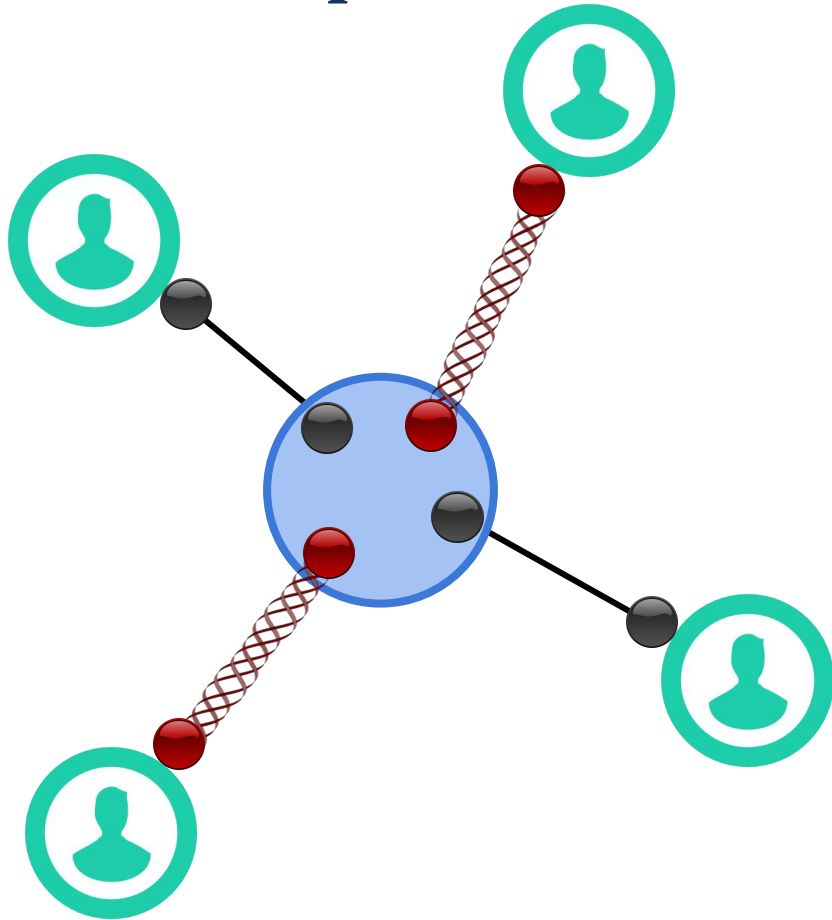
- perform a 
- store in memory
- discard

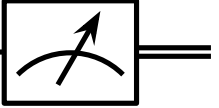
# Switch Operation



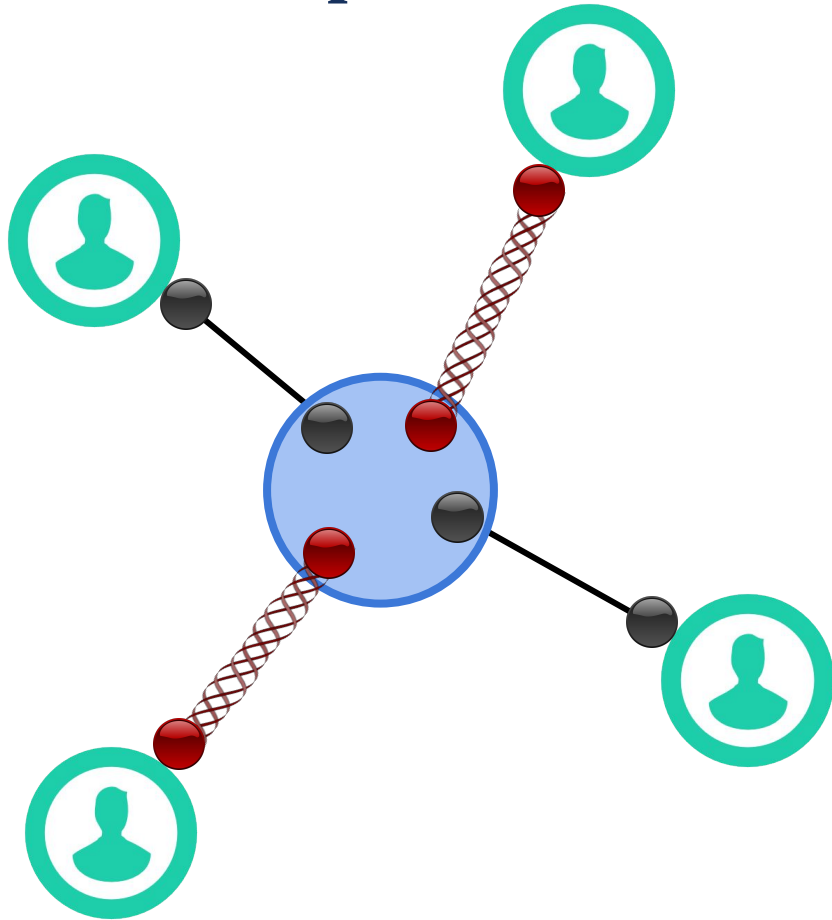
- perform a 
- store in memory
- discard


# Switch Operation



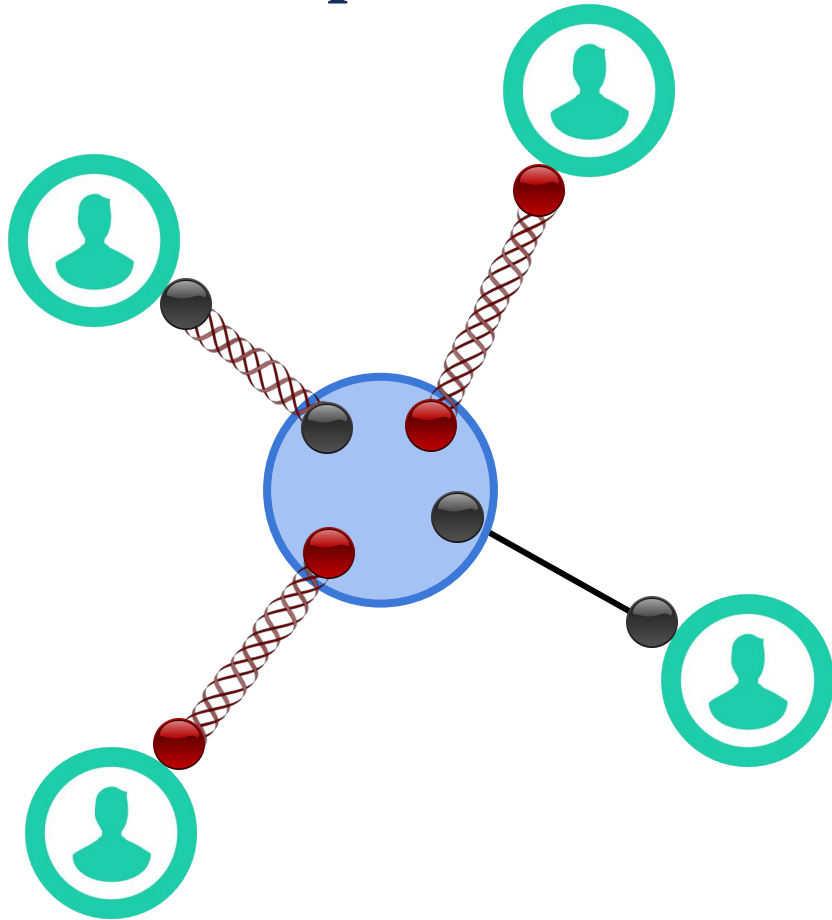
- perform a 
- store in memory
- discard


# Switch Operation



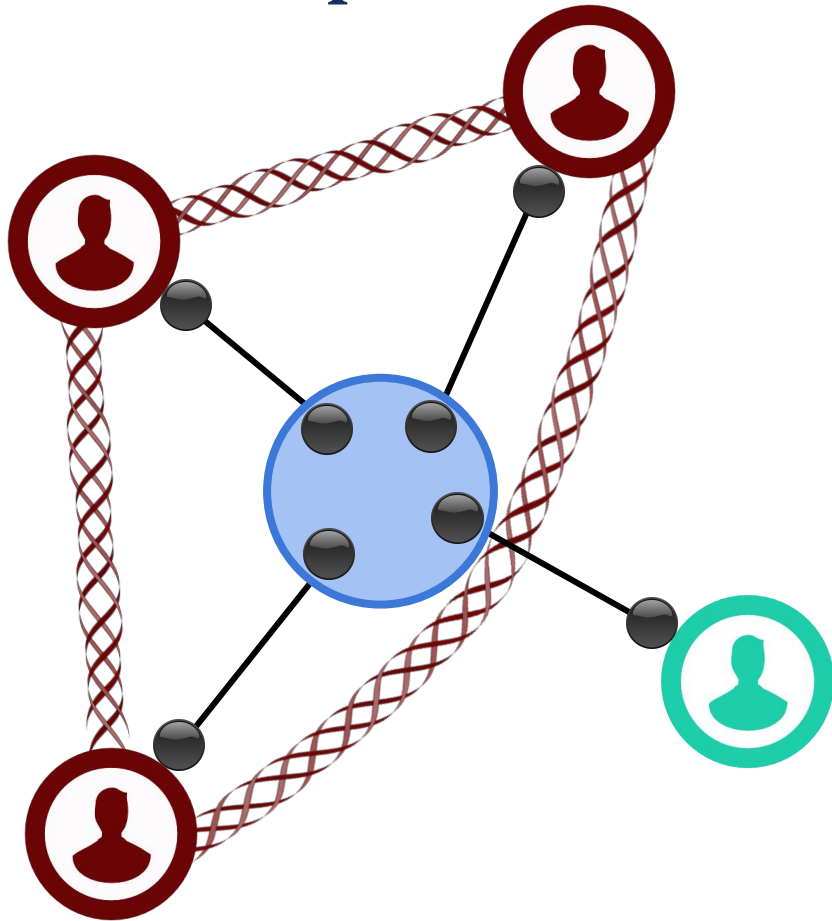
- perform a 
- store in memory
- discard
- wait


# Switch Operation



- perform a 
- store in memory
- discard
- wait

# Switch Operation



- perform a 
- store in memory
- discard
- wait

# The Model

Recall:

Link-level entanglement generation: **Poisson** process

*Successful* link-level entanglement generation rate  $\mu$

Coherence time: **exponential** r.v.

Decoherence rate  $\alpha$

Interested in **capacity** of switch

→ any set of  $n=2/3$  of users want to share an entangled state

→ Bell pairs used as soon as there are enough to generate an  $n$ -partite entanglement

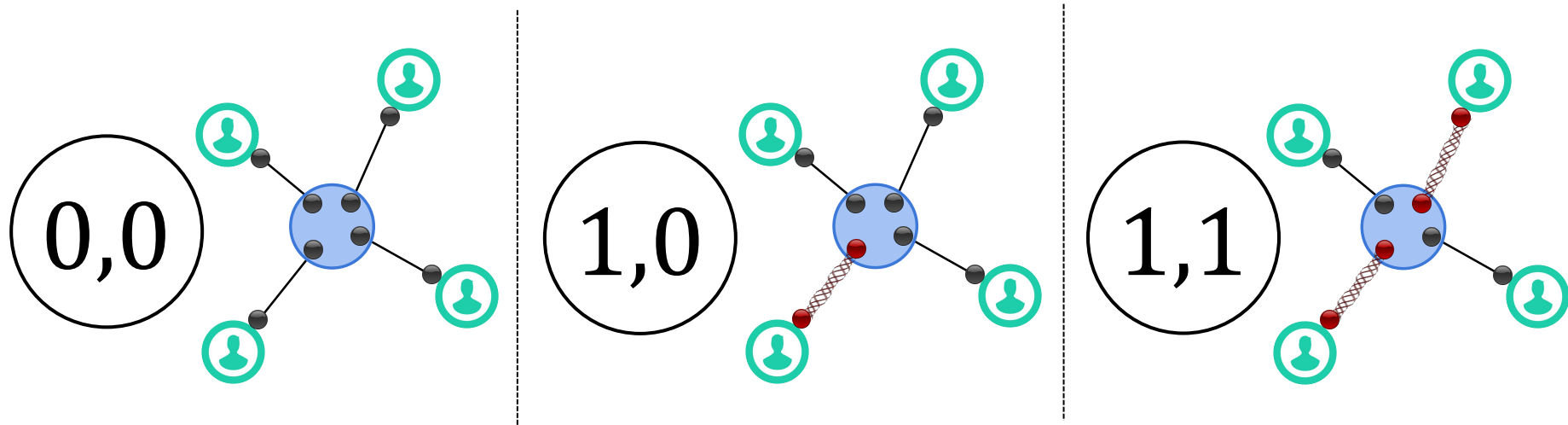
Can model using a continuous-time Markov chain.



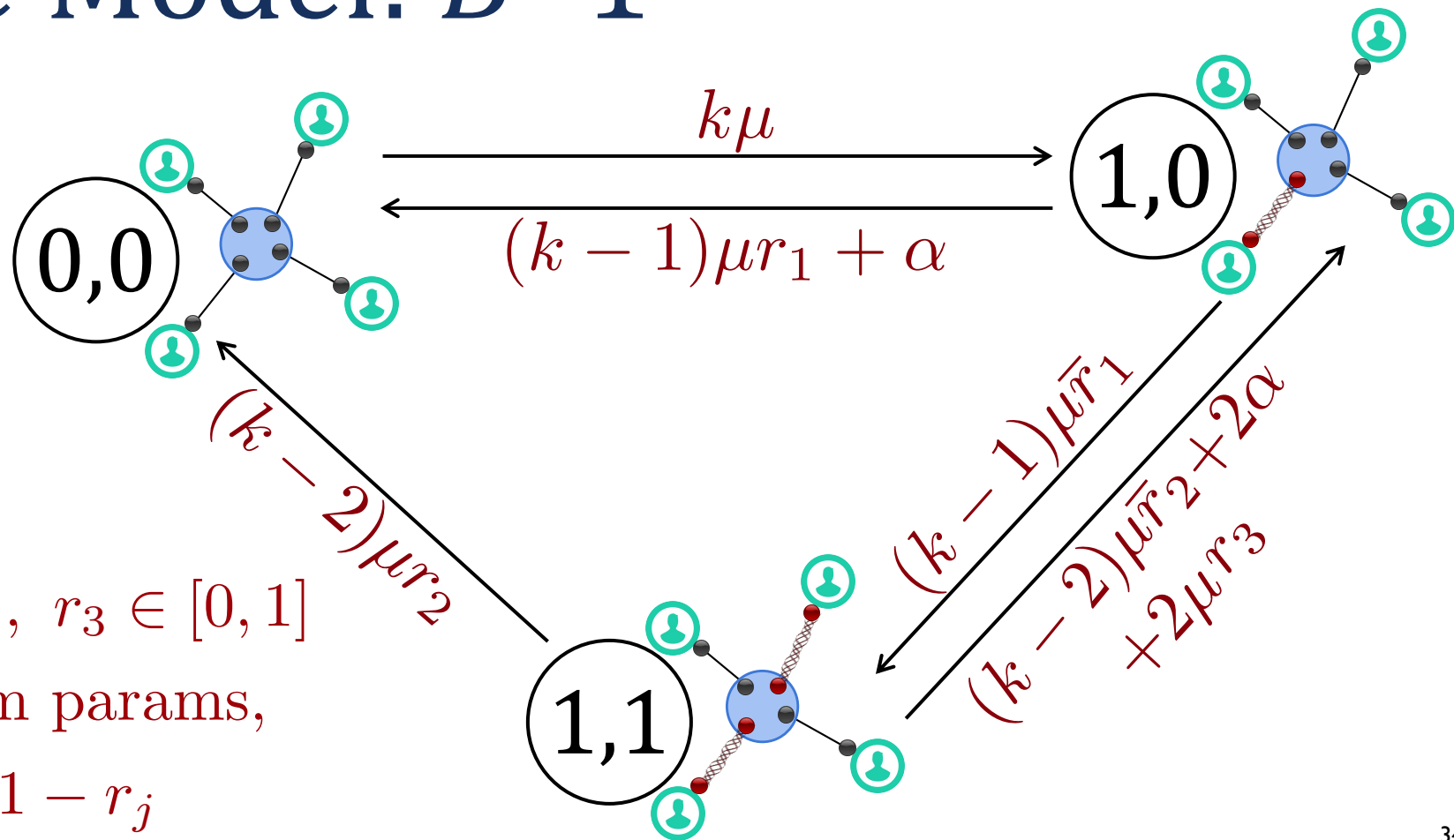
# The Model: $B=1$ state space

Bell pairs used as soon as there are  
enough for a swap

→ up to 2 links will store Bell pairs

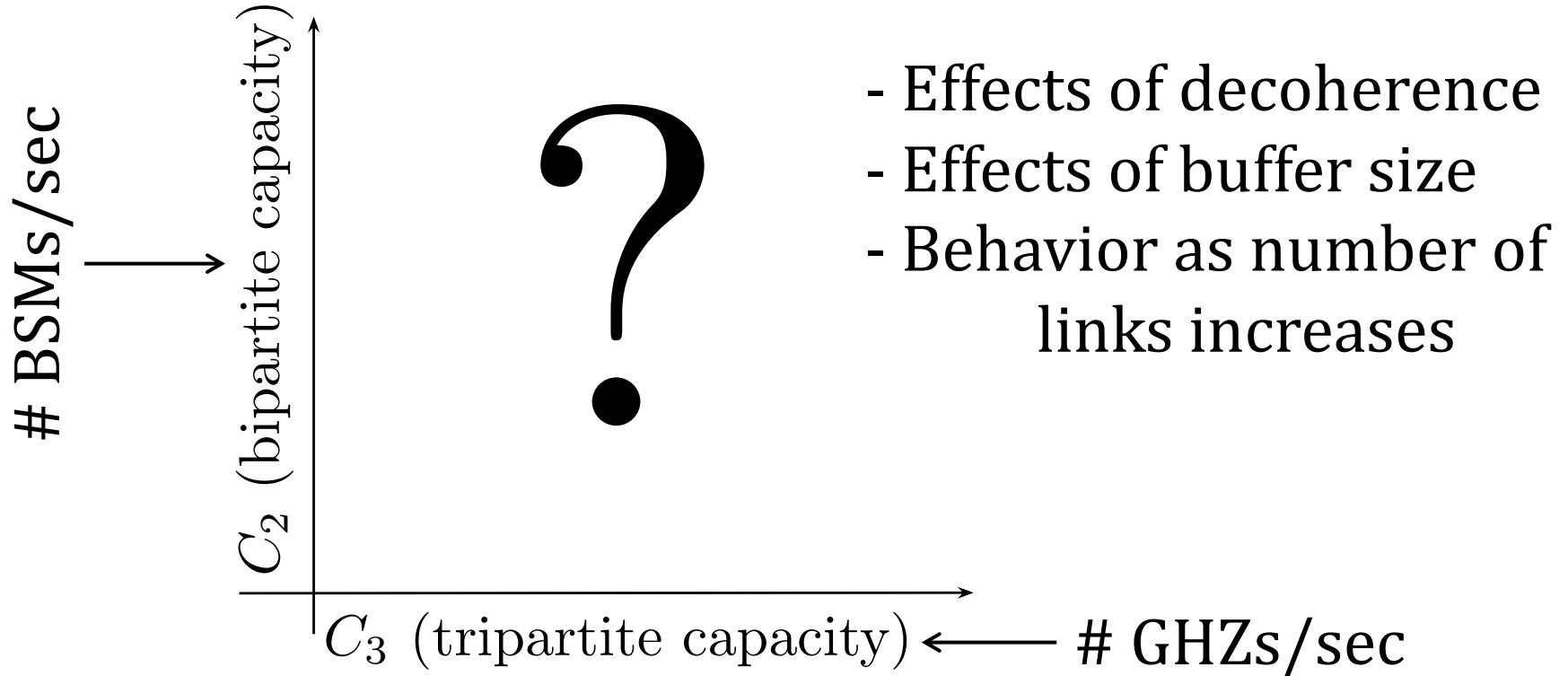


# The Model: $B=1$

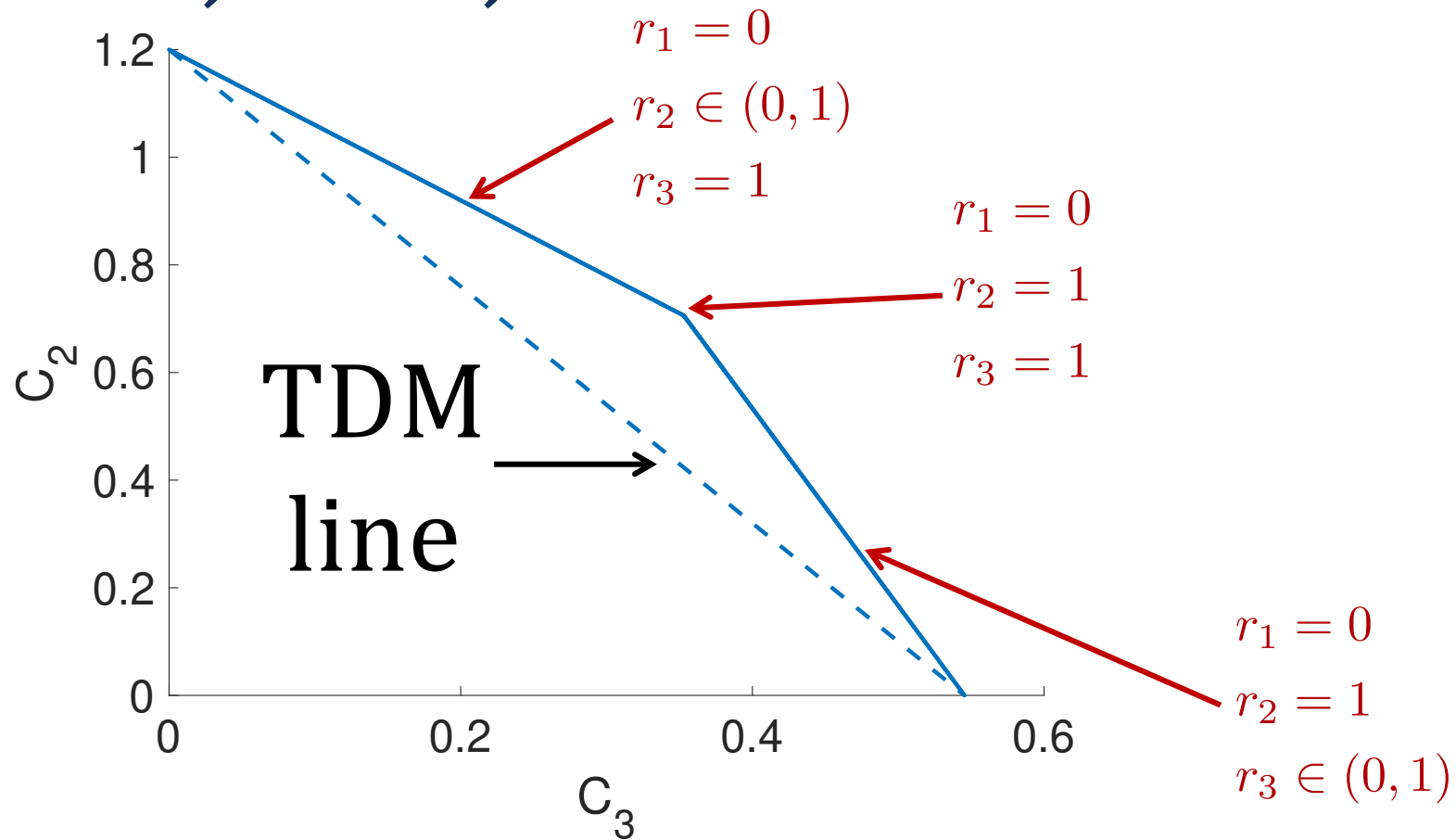


Q1: What is the capacity region of the switch?

Q2: Are there better policies than TDM?

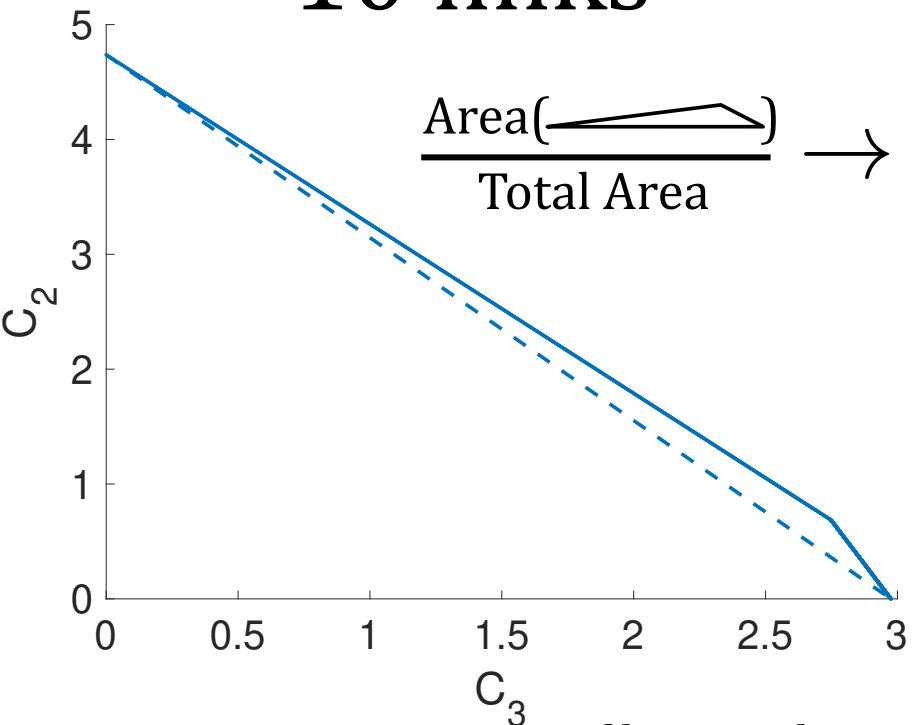


# $k=3, B=1$ , no decoherence



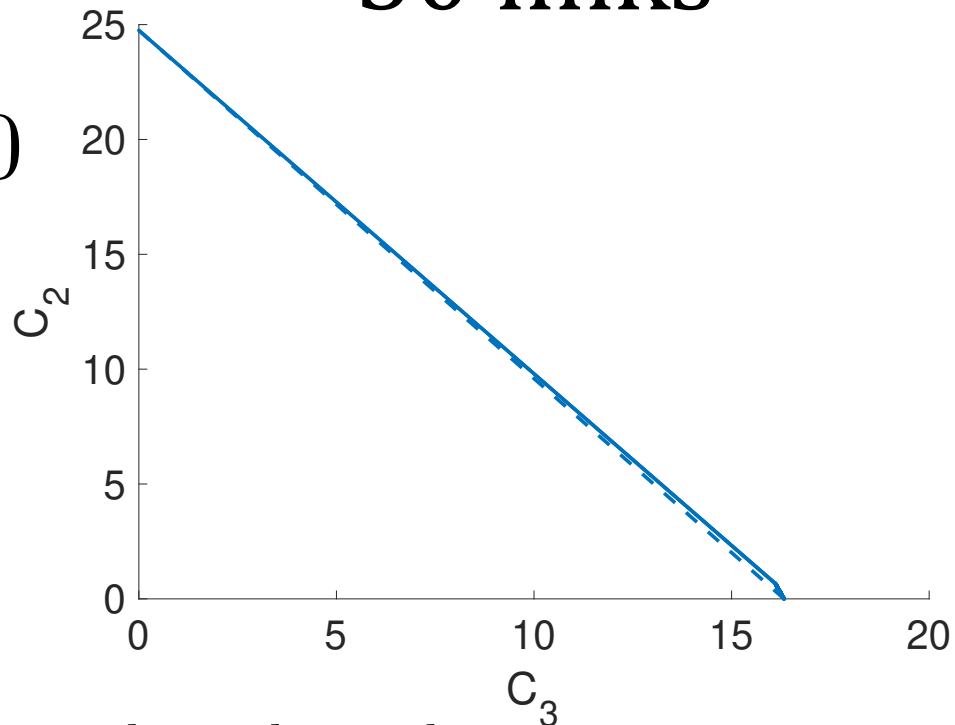
# $B=1$ , no decoherence

## 10 links



$$\frac{\text{Area}(\text{triangle})}{\text{Total Area}} \rightarrow 0$$

## 50 links



Fraction of better-than-TDM policies diminishes.

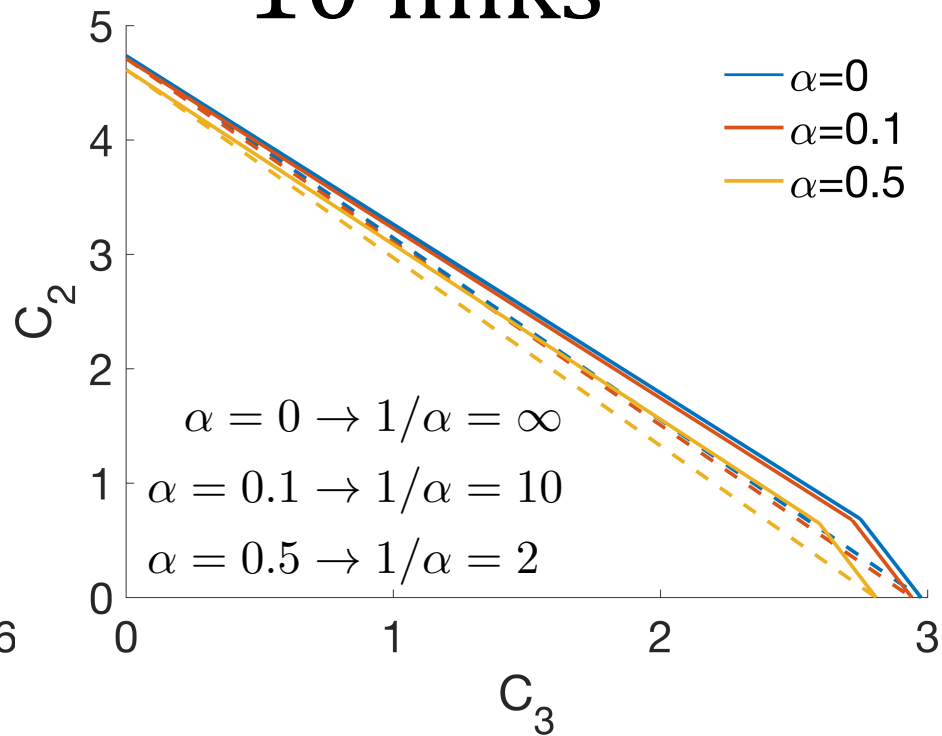
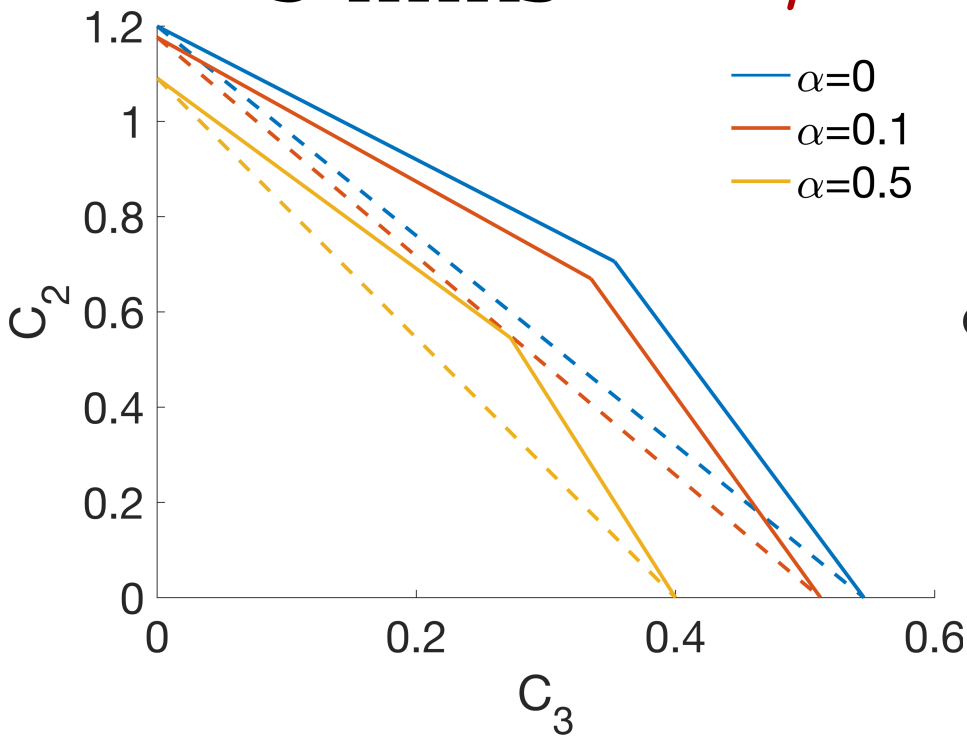
Similar results for  $B=2$  systems.

# Effects of Decoherence, $B=1$

3 links

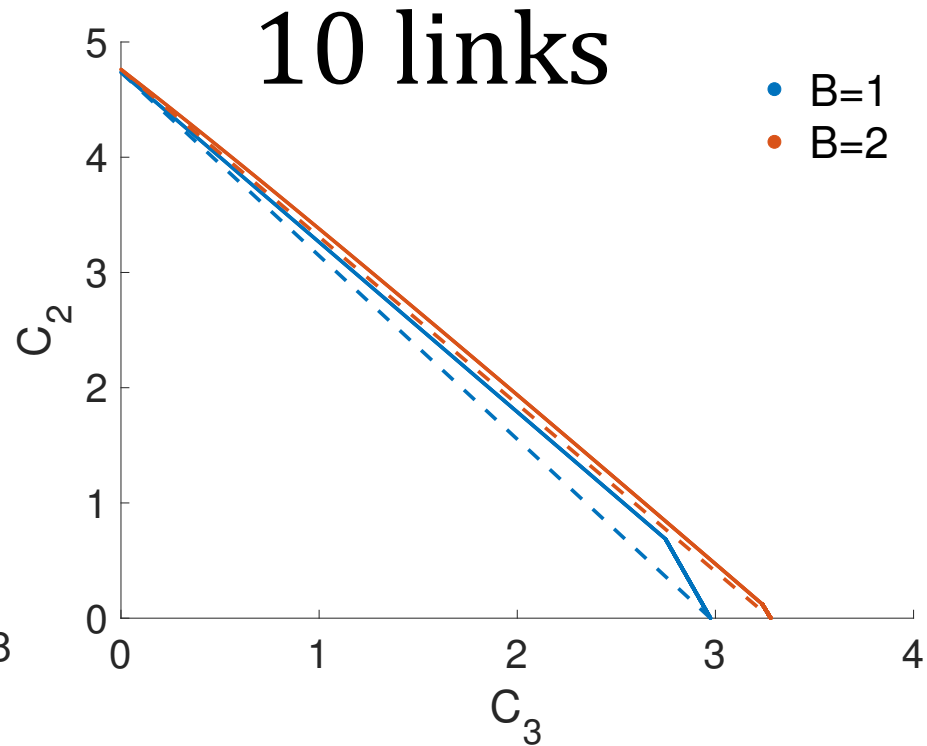
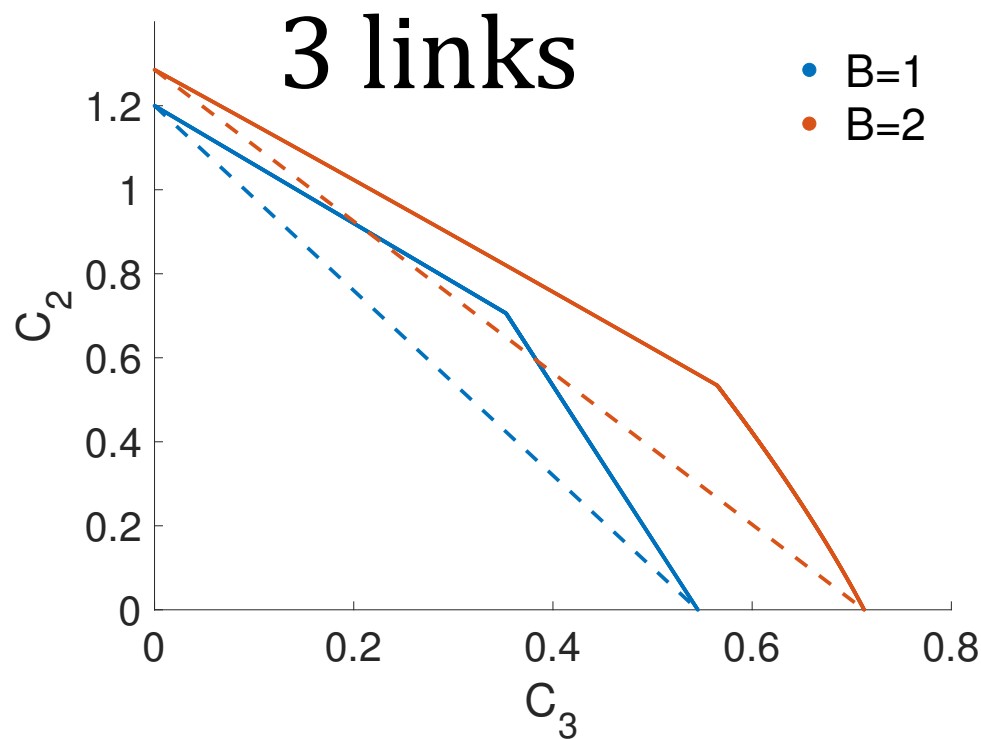
$\mu = 1$

10 links



Similar results for  $B=2$

# Effects of Buffer Size



1. Advantages of extra buffer diminish with  $k$
2.  $C_3$  benefits more from extra buffer than  $C_2$

# Summary & Future Directions

- Bipartite/tripartite entanglement switching problem
- Modeled as a CTMC
- Explored randomized switching policies
  - There is always a better policy than TDM
  - Advantages diminish as number of users grows
  - Analytical proofs for buffer-size one systems
- Extensions
  - Non-identical links
  - Non-unit state fidelities
  - Larger buffer sizes
- User requests & optimality of scheduling policies



# Thank You!

## Questions?



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