On the Exact Analysis of an Idealized Quantum Switch

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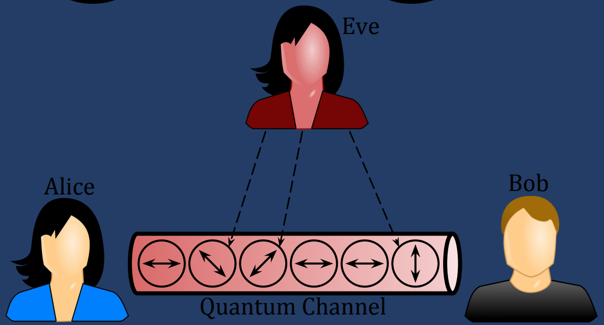
Performance 2020

Distributed Quantum Systems

- Cryptography
  - Quantum Key Distribution
  - Quantum Secret Sharing
  - Byzantine Agreement
- Scientific
  - Clock Synchronization
  - Long Baseline Interferometry
  - Bell Tests
- Distributed Quantum Computation
  - Using small quantum registers
  - Blind Quantum Computation



C. Bennett & G. Brassard, 1984;
A. Ekert 1991

D. Gottesman,
Phys Rev Lett 2012

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Quantum communication offers functionality that is not achievable classically.

This is often achieved by exploiting the properties of entangled quantum states.

Talk Outline

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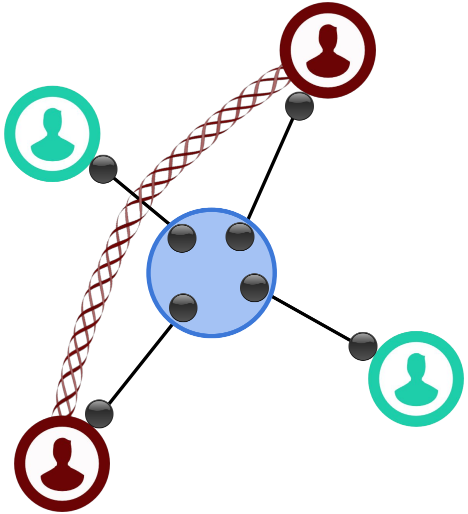
Entanglement switching

- Problem statement
- Switch description
- Prior work
- Analysis
- Results



Summary & Future Directions

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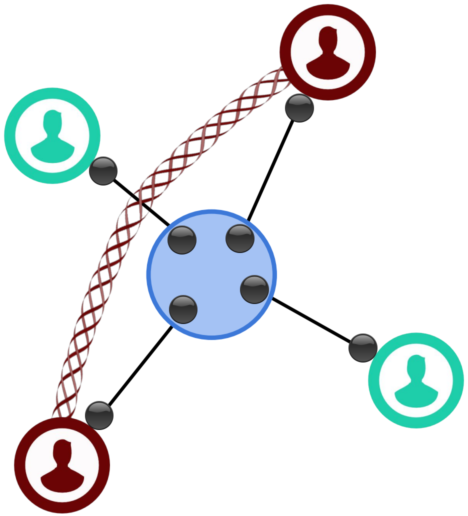
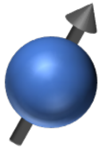


Summary & Future Directions

The Basics: Quantum Bits

- qubit: quantum analogue of a bit
- represented by a two-level QM system
    - *e.g.*, photon polarization, electron spin



$$\longleftrightarrow |0\rangle \equiv \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\longleftrightarrow |1\rangle \equiv \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

- a qubit can be in ***superposition***

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

$$\alpha, \beta \in \mathbb{C},$$

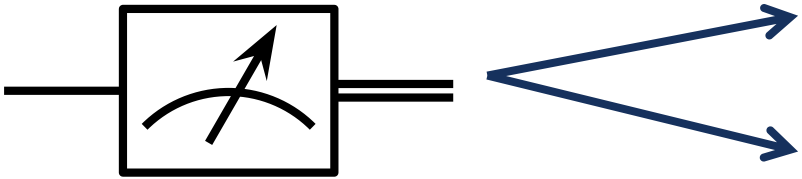
$$|\alpha|^2 + |\beta|^2 = 1$$

The Basics: Entanglement

Two or more qubits are said to be entangled when the state of one cannot be described independently from the state of the other(s).

Example: a Bell state, single most important resource for quantum communication

$$\frac{|00\rangle + |11\rangle}{\sqrt{2}} = \frac{|0\rangle_A \otimes |0\rangle_B + |1\rangle_A \otimes |1\rangle_B}{\sqrt{2}}$$



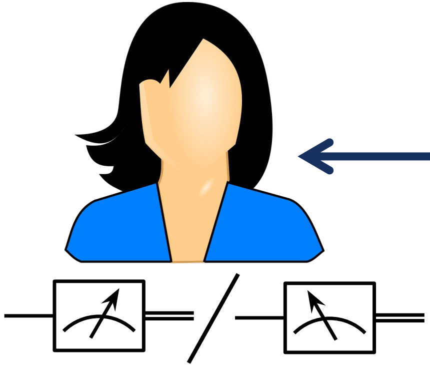
$$|0\rangle_A \to |0\rangle_B$$

$$|1\rangle_A \to |1\rangle_B$$

Quantum Key Distribution: the E91 protocol

Goal: create One-Time Pad – only provably
secure means of encryption

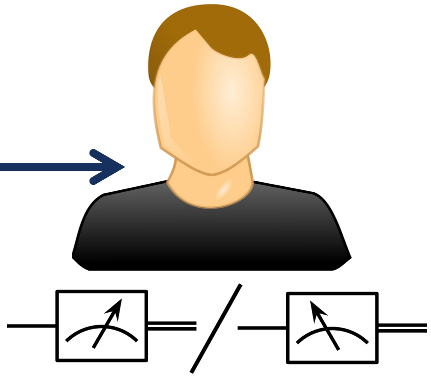
Alice



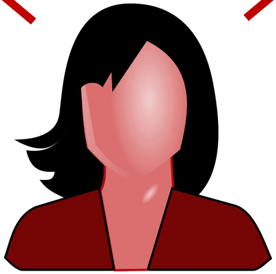
Bell pair

$$\frac{|0\rangle_A |0\rangle_B + |1\rangle_A |1\rangle_B}{\sqrt{2}}$$

Bob



Eve



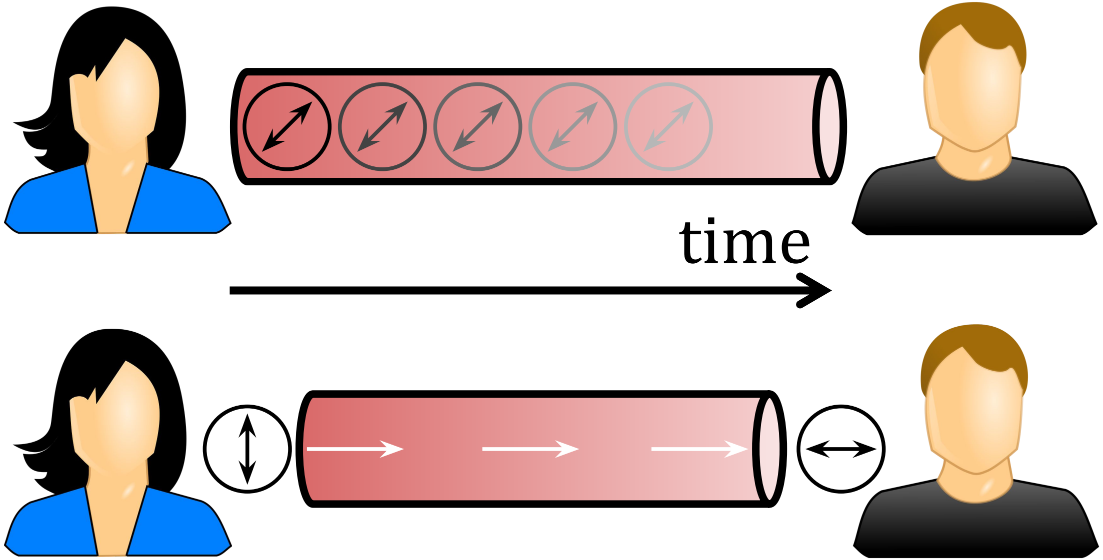
all-powerful adversary

Ex:

Alice	Bob	Prob.
0	0	1/2
1	1	1/2

can **detect eavesdropping**
using statistical methods
(Bell-type inequalities)

Transporting a Quantum State



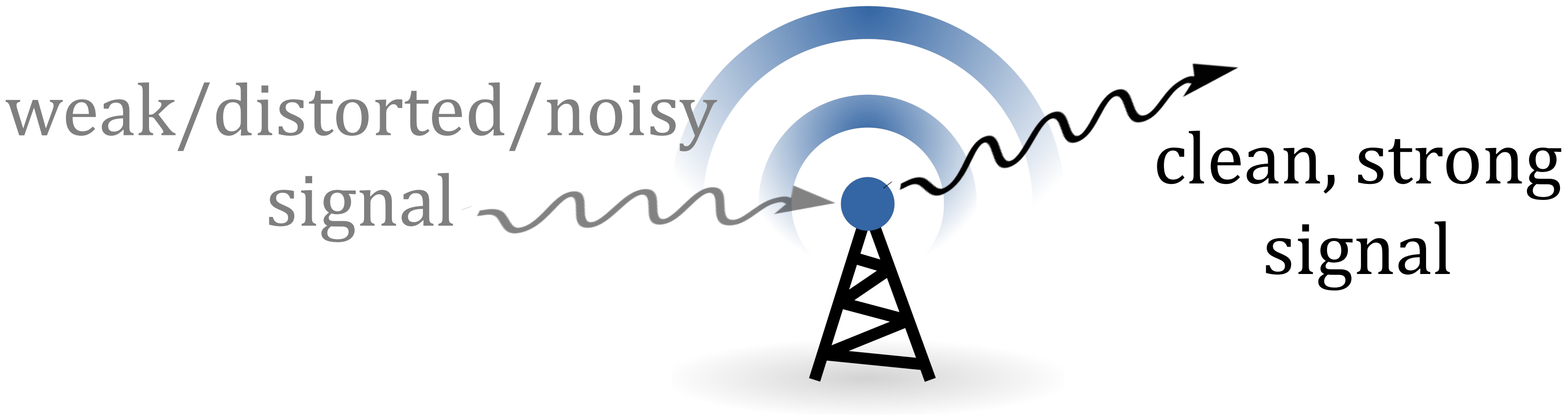
fiber loss coefficient

$$P(\text{photon makes it to receiver}) \sim e^{-\alpha L}$$

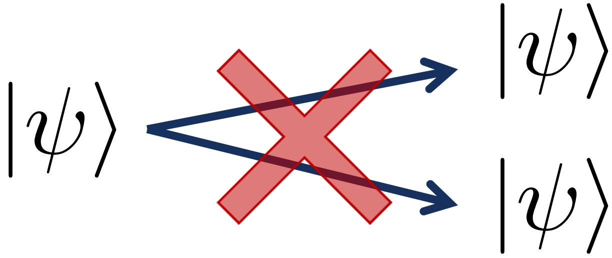
fiber length

longer channel → higher probability of loss

In a **classical** network...

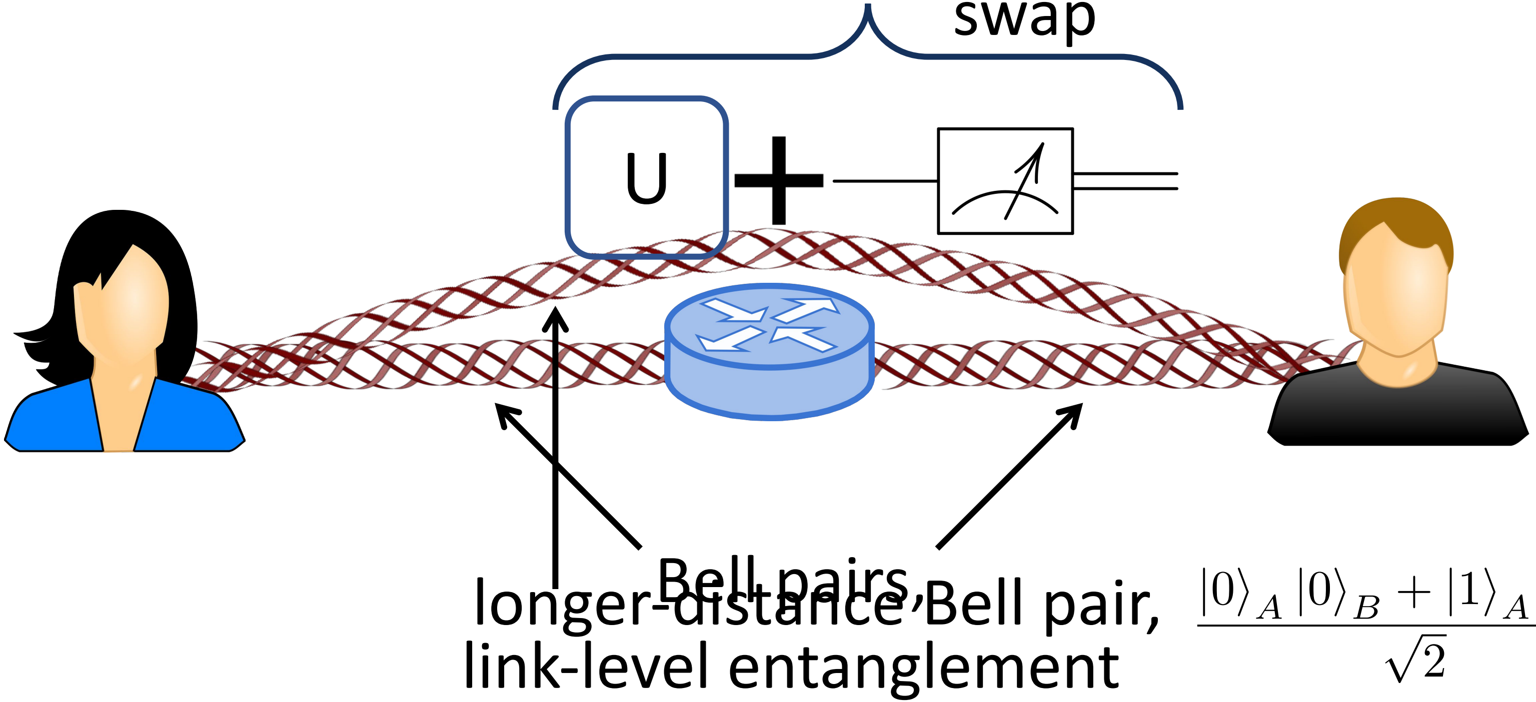


On a **quantum** channel...



→ No-Cloning Theorem

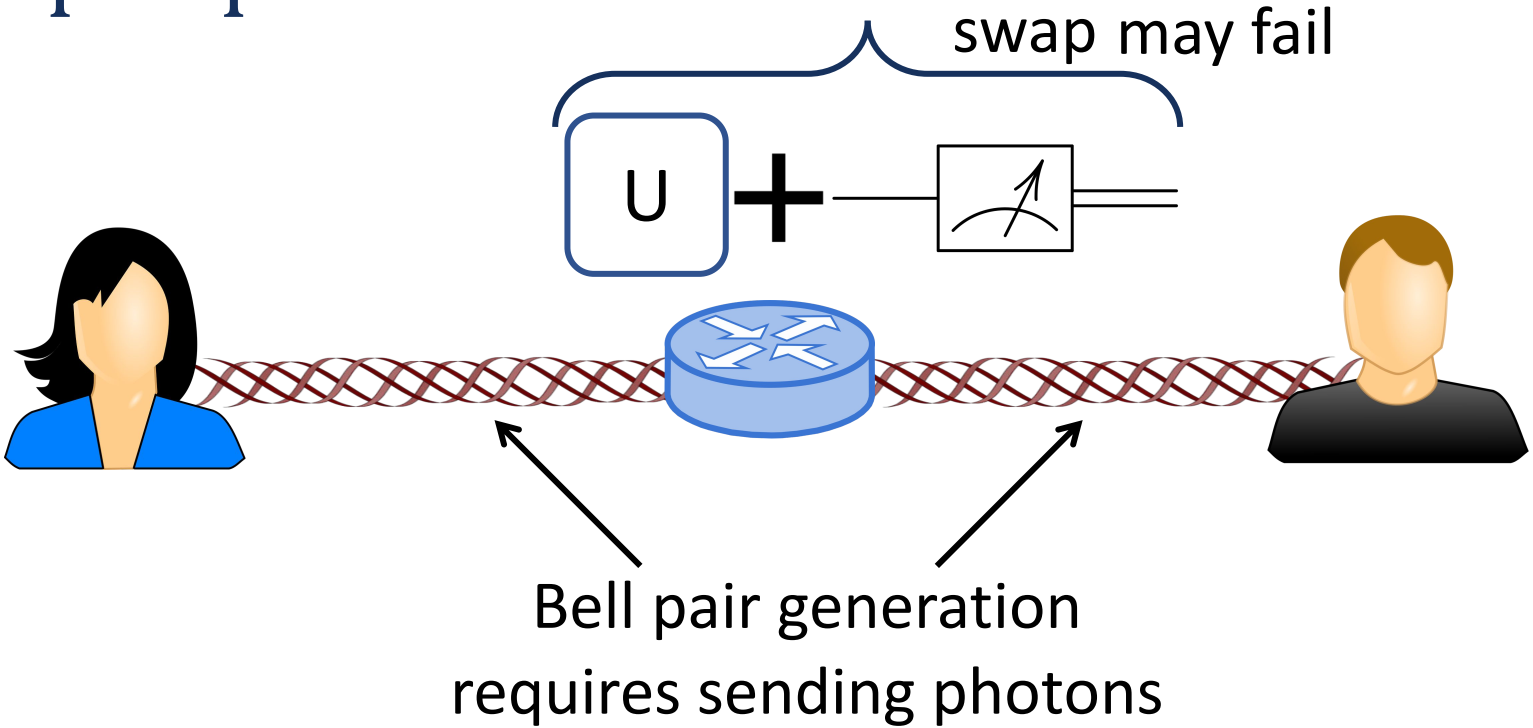
A simple quantum network



$$\frac{|0\rangle_A |0\rangle_B + |1\rangle_A |1\rangle_B}{\sqrt{2}}$$

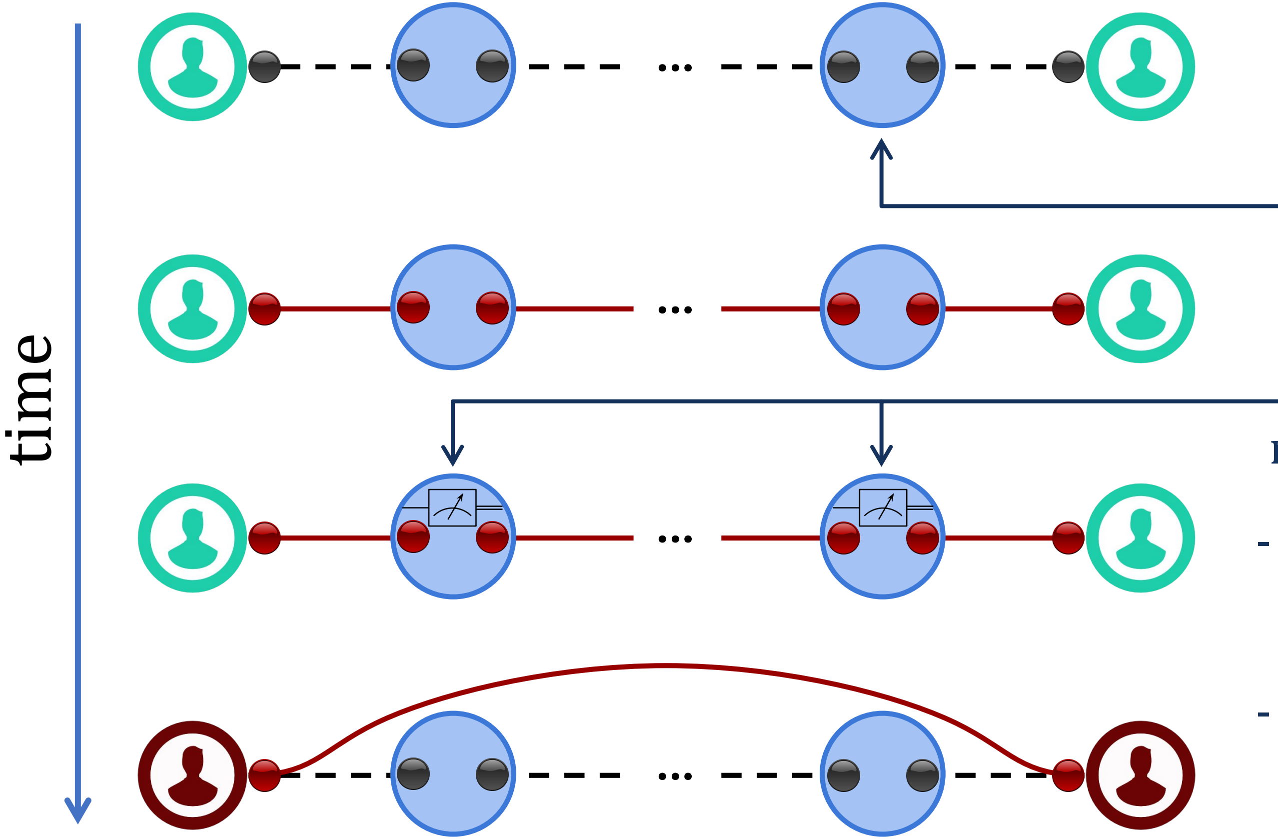
- Teleportation (consumes one Bell pair)
- Use directly, *e.g.*, in E91 or similar protocols

A simple quantum network



$$P(\text{successful link-level entanglement}) \sim e^{-\alpha L/2}$$

A Quantum Network



quantum switch – a device fundamental to quantum networks

Bell state measurements

- What is the capacity of a quantum switch?
- How many qubits are needed?

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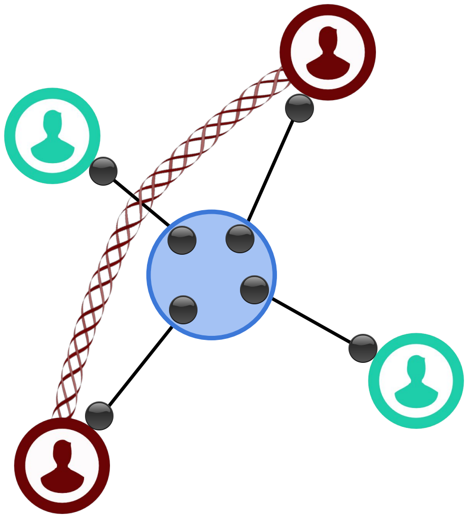

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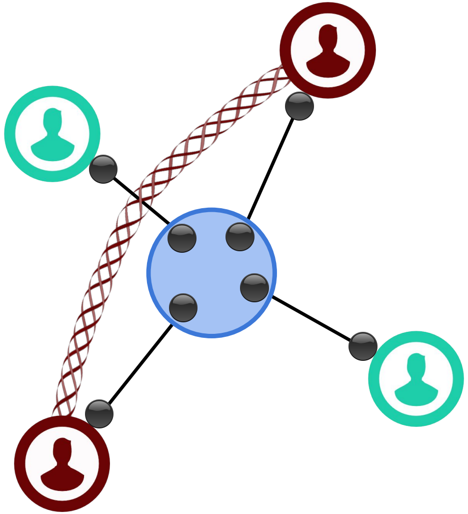
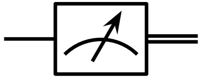


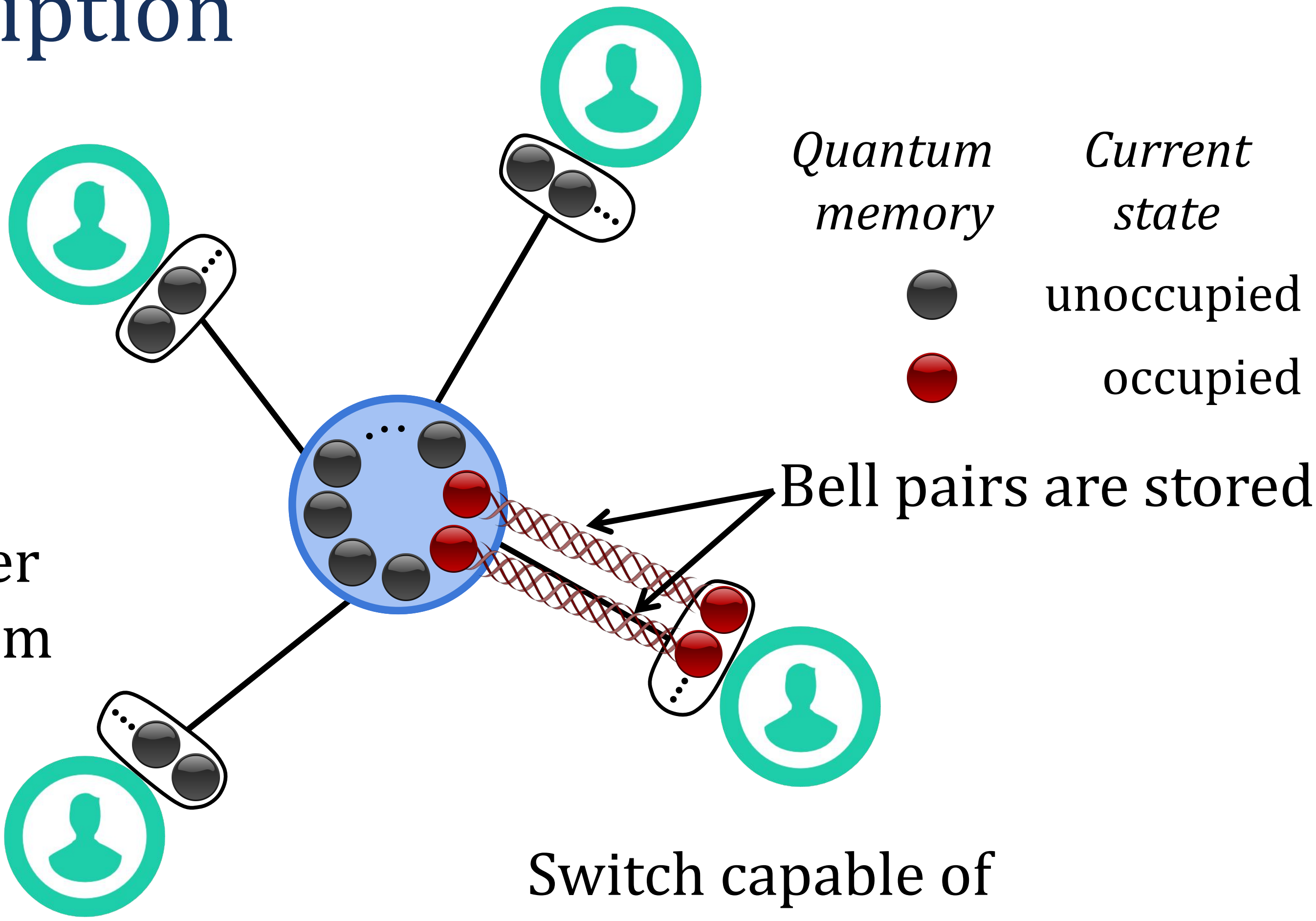
Summary & Future Directions

Switch Description

Users do not limit performance

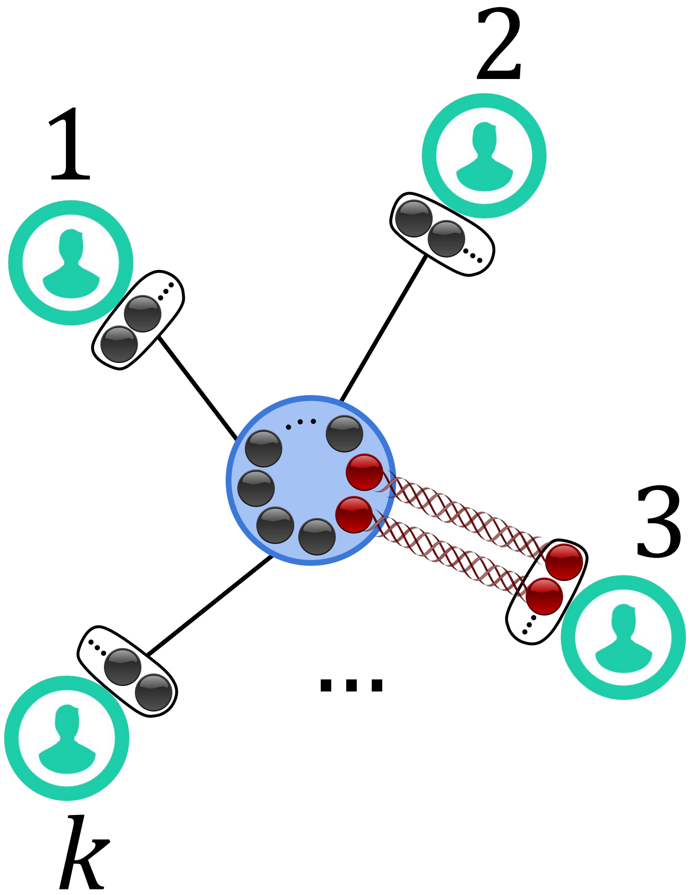
Switch is equipped with infinite number of noiseless quantum memories (buffer)

Switch can perform projective 



Switch capable of classical communication

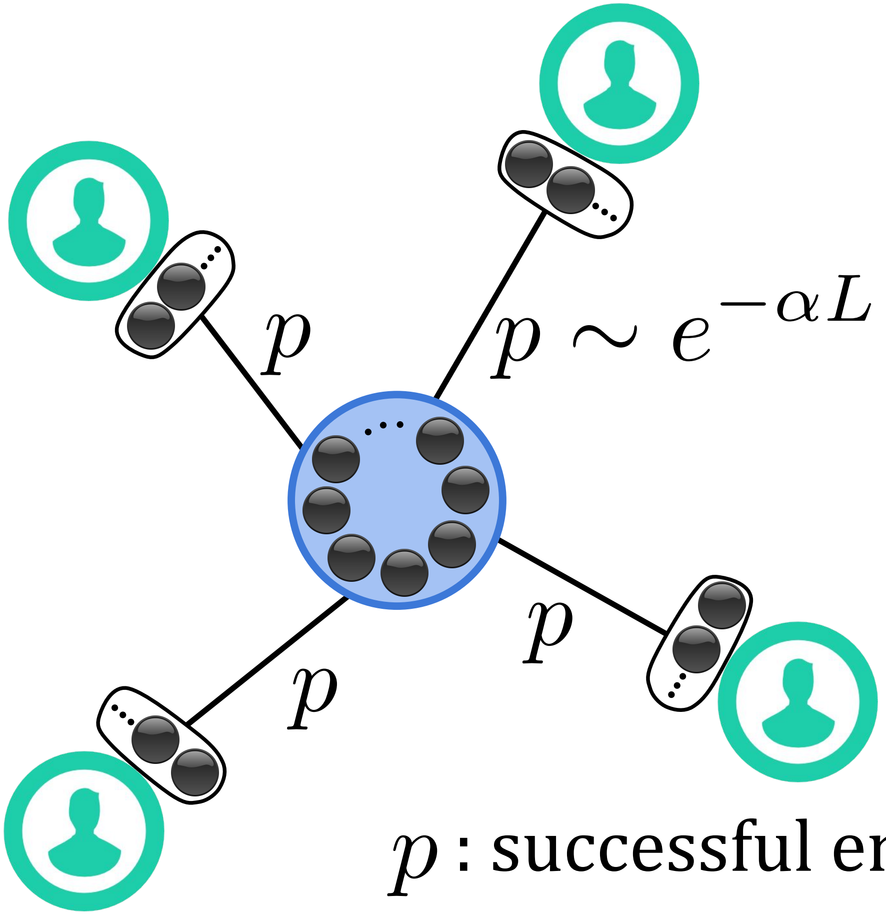
Switch Operation



Switch serves entangled states to *k* users

1. Bell pairs over individual links
2. switch performs a swapping operation, resulting in end-to-end Bell state for a pair of users

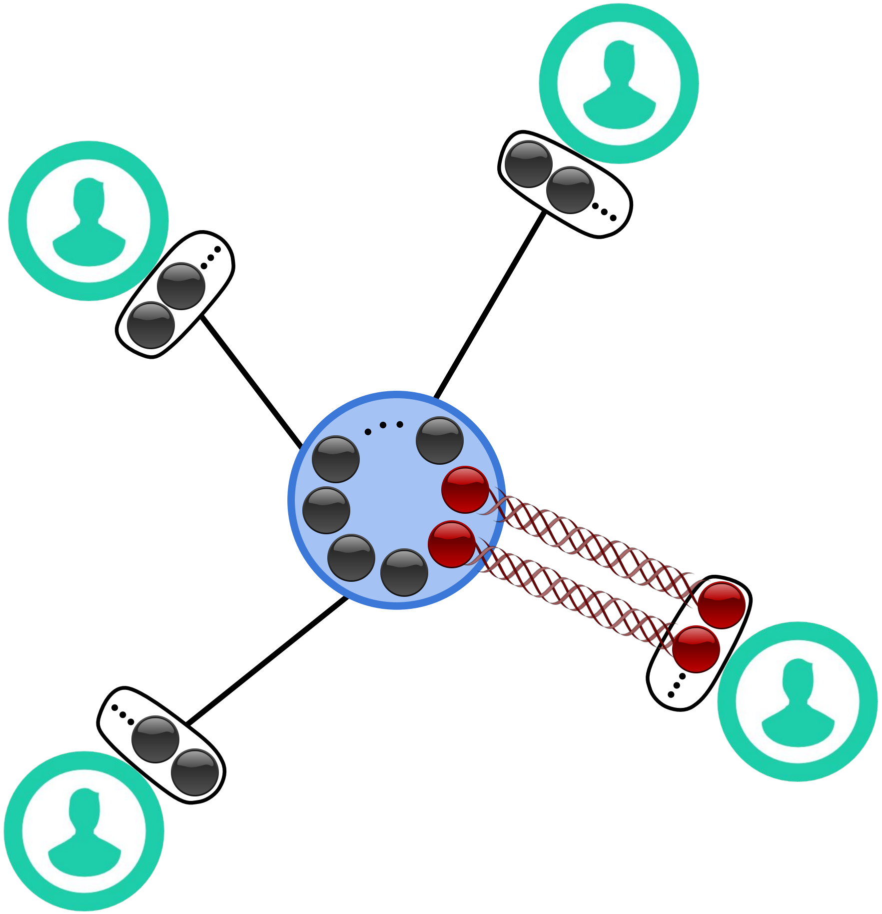
Modeling Link-Level Entanglement Generation



1. link-level entanglement generation
   - assume links are identical (equal length)
   - every τ seconds, all links attempt entanglement generation

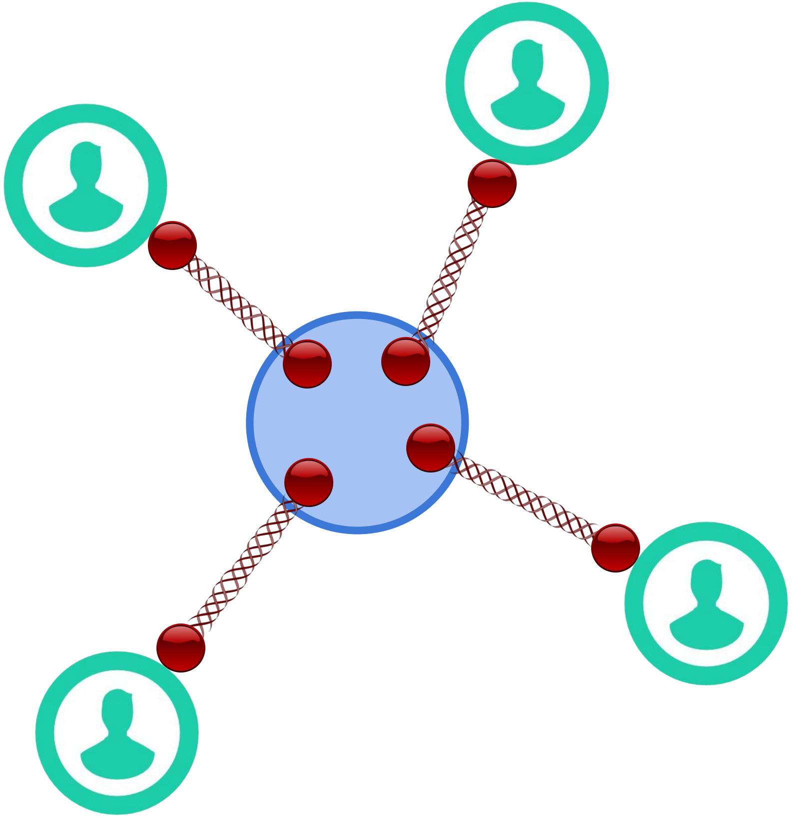
p : successful entanglement generation rate

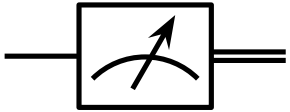
Switch Operation



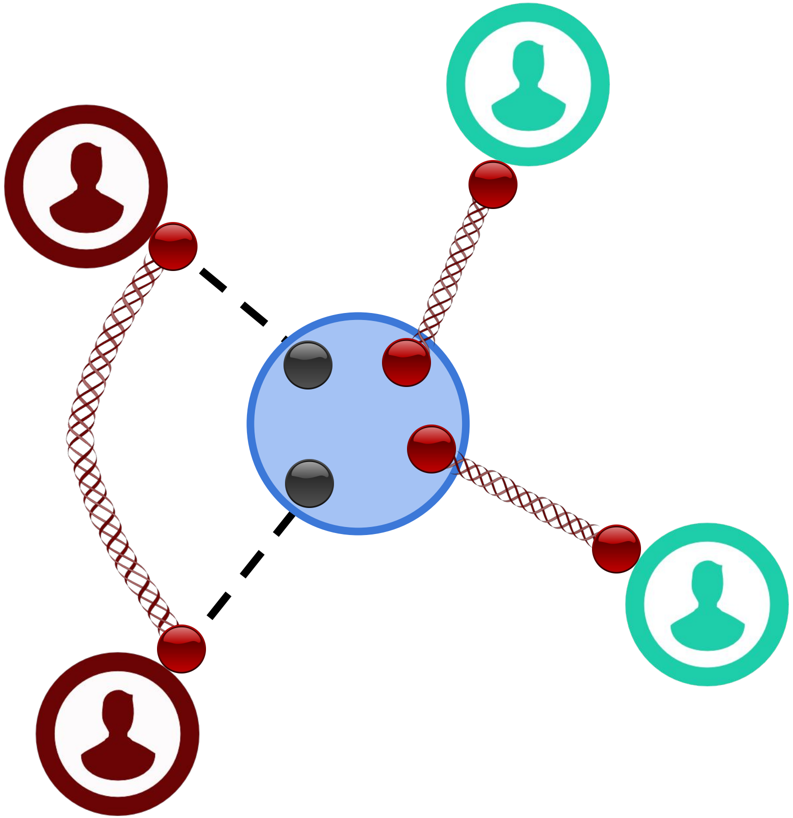
1. link-level entanglement generation
2. swapping, according to user demands and switching policy

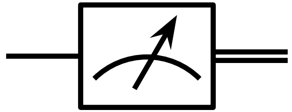
Switch Operation: beginning of time slot 1



- perform a 
- store in memory
- wait

Switch Operation: time slot 1, cntd.



- perform a 
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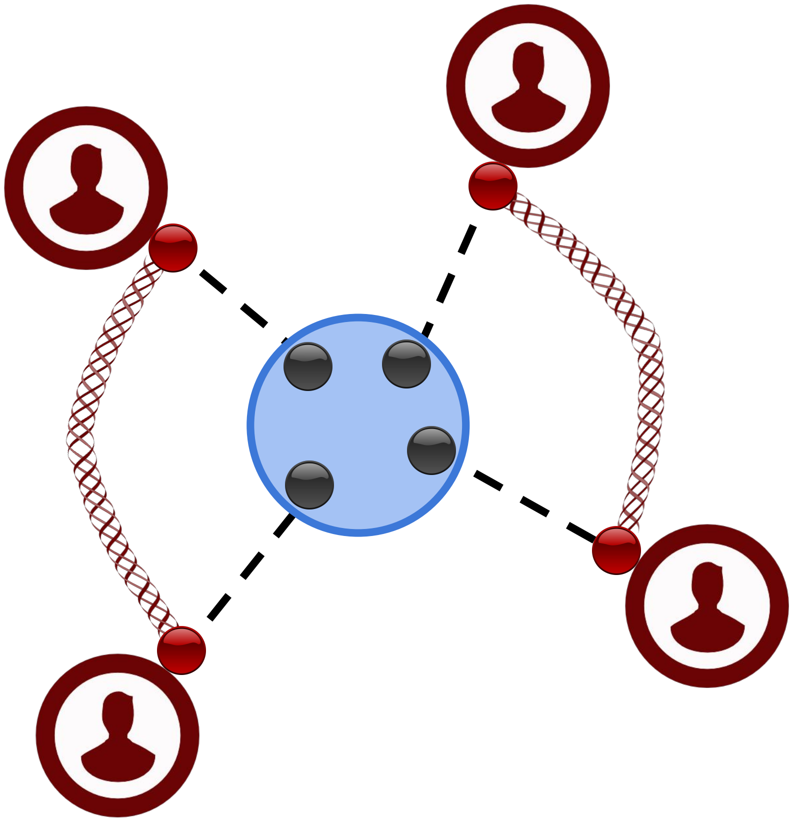
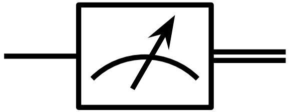
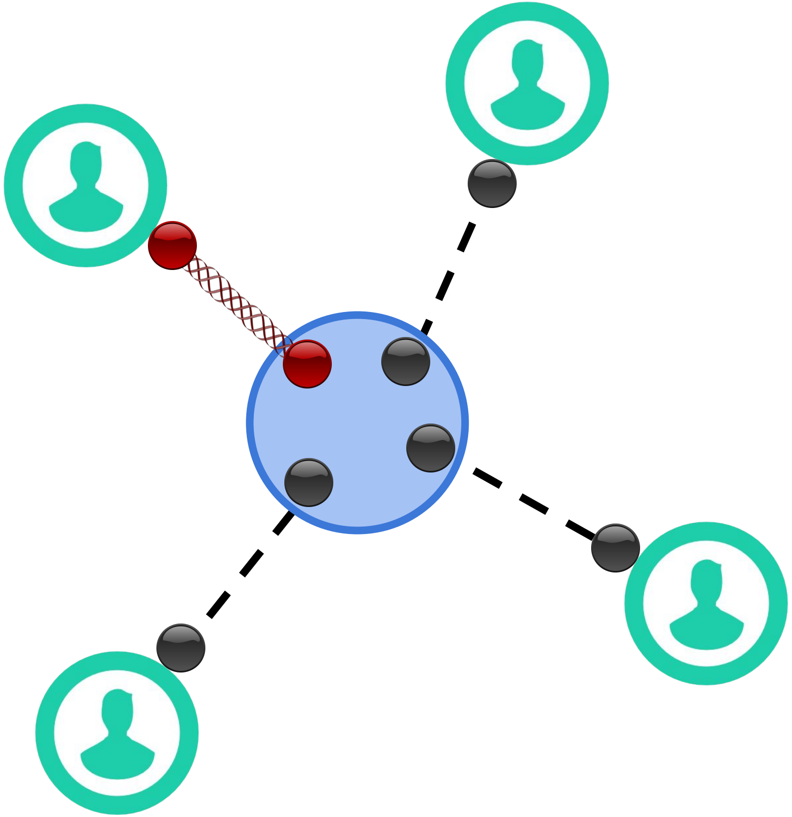
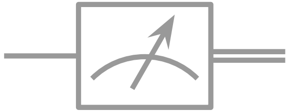
Switch Operation: time slot 1, cntd.



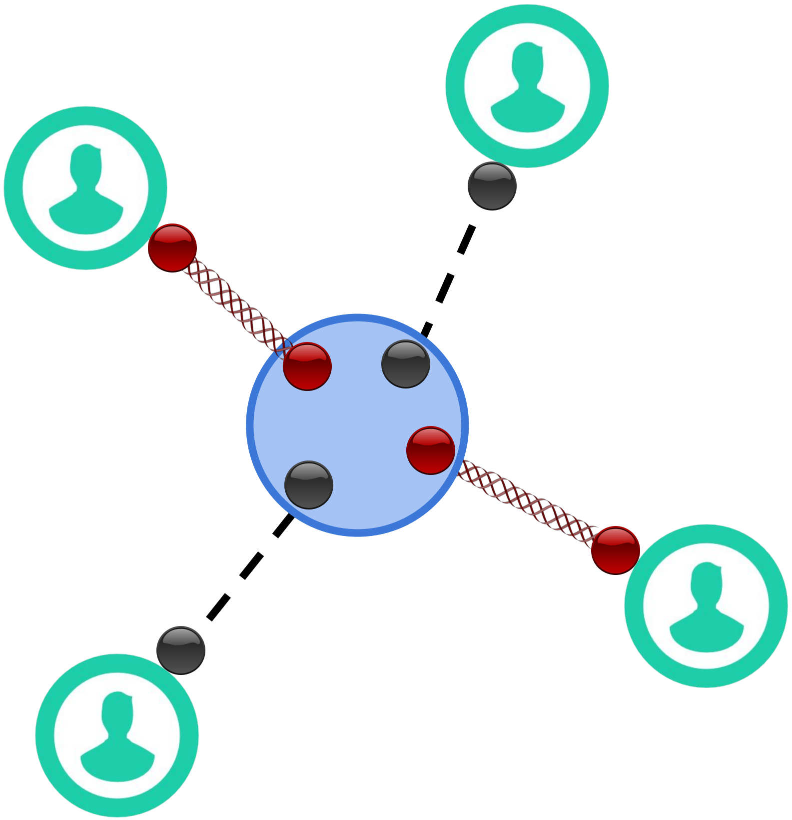
- perform a 
- store in memory
- wait

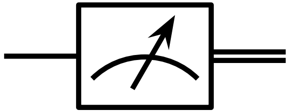
Switch Operation: beginning of time slot 2



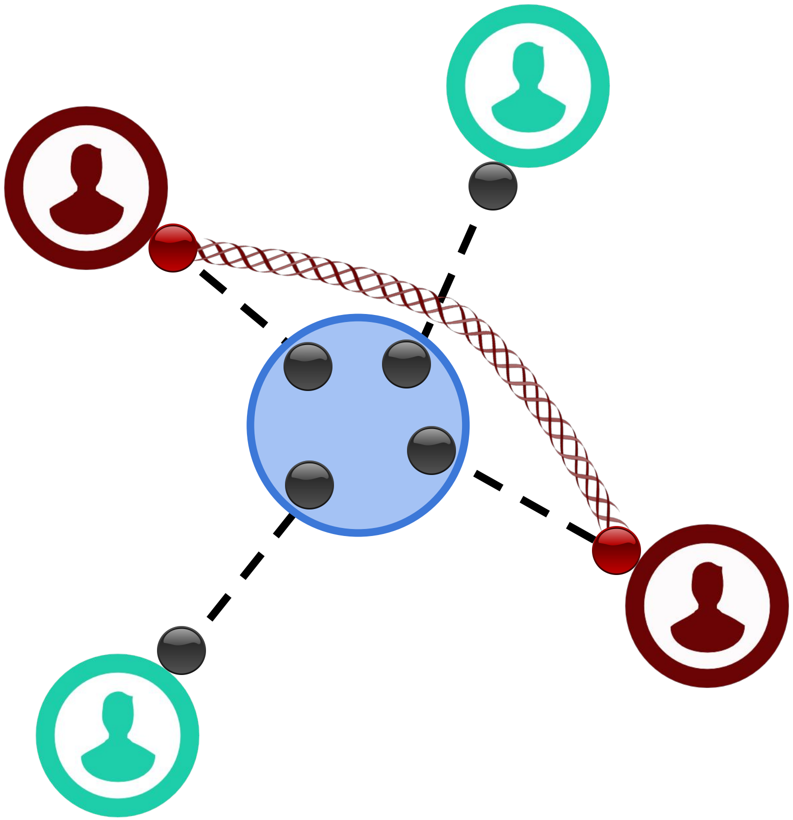
- perform a 
- store in memory
- wait

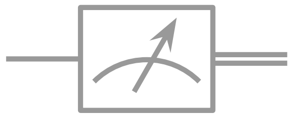
Switch Operation: beginning of time slot 3



- perform a 
- store in memory
- wait

Switch Operation: beginning of time slot 3



- perform a 
- store in memory
- wait

*All quantum states assumed to have unit fidelity to corresponding ideal pure states

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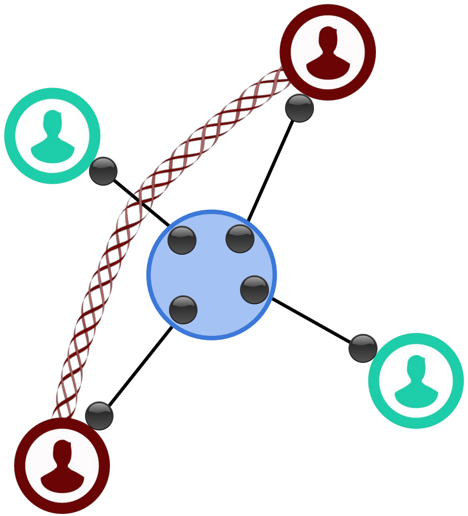

Summary & Future Directions

Prior Work

- Continuous-time Markov chain for bipartite switching (Vardoyan *et al.*, ACM PER 2019)
  - Model link-level entanglement generation as a Poisson process
  - Easily extendable to finite quantum memories, heterogeneous links, quantum state decoherence
- Continuous-time Markov chain for multipartite switching (Nain *et al.*, SIGMETRICS 2020/POMACS)
  - Focuses on idealized scenario (infinite memory, no decoherence, identical links)
  - Reduces to ACM PER 2019 for bipartite case

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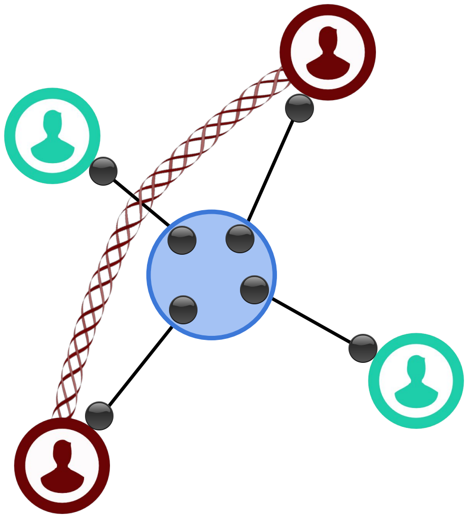

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Preliminaries

Definitions: when in steady state (if it exists)

- Capacity *C* - # end-to-end entangled states served per time slot
- Expected number of stored qubits at switch *E[Q]*

Interested in **capacity** of switch – max rate of switching

→ any pair of users wish to share an entangled state

→ Bell pairs used as soon as there are enough to generate an end-to-end Bell pair

→ Oldest Link Entanglement First (OLEF)

The Model

Link-level entanglement generation: Bernoulli process

Successful link-level entanglement probability p

Coherence time: infinite

Swap success probability q

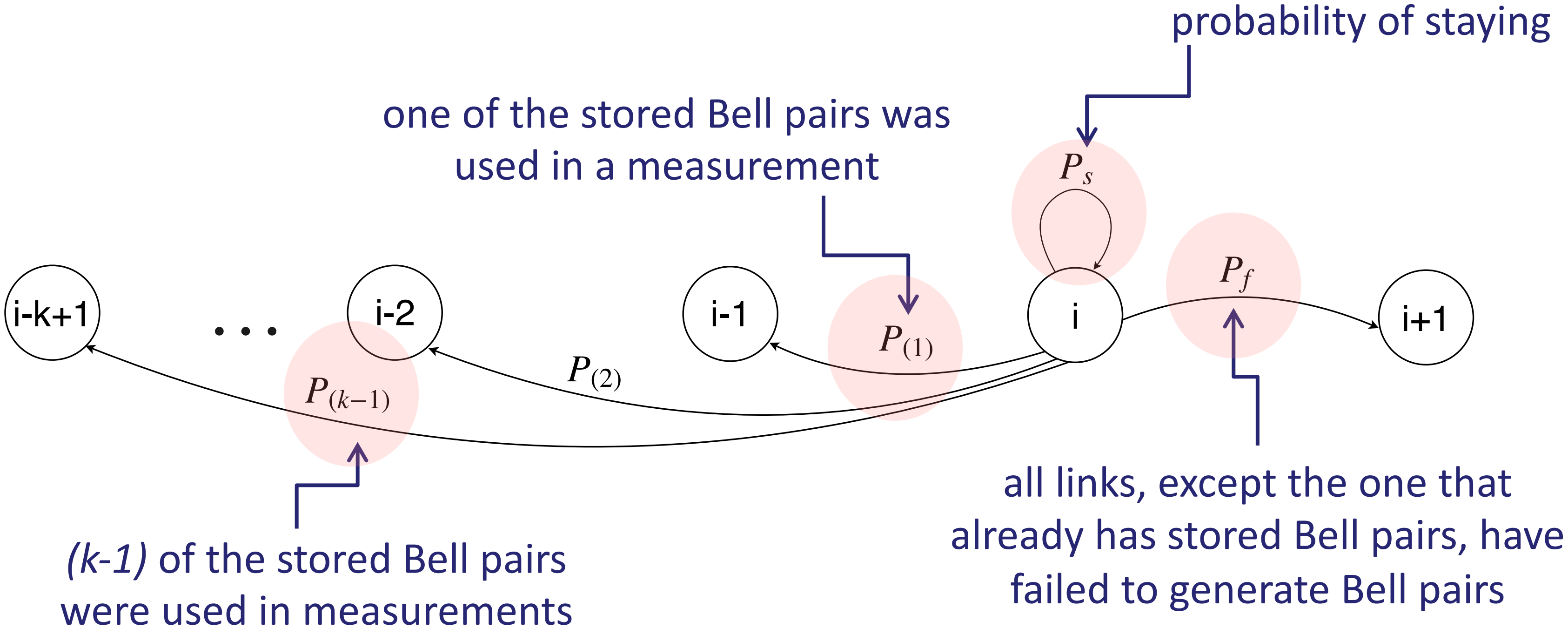
Can model using a discrete-time Markov chain with state space

$$\Omega := \{0, 1, 2, \ldots\}$$

# of stored qubits for link with Bell pairs

at most one link will store qubits at any given time

Transitions from state *i*



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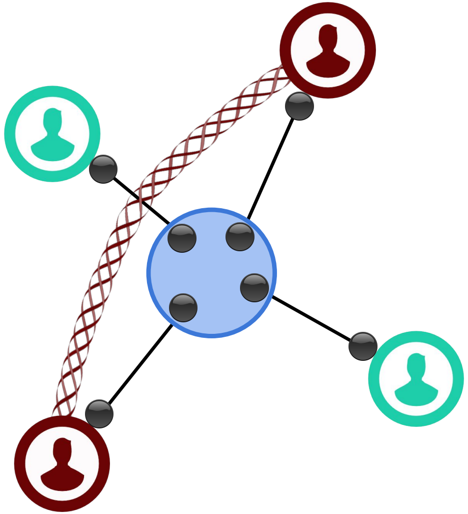


Summary & Future Directions

Analytical Results

The system is stable if and only if $k > 2$

When the system is stable, the capacity is

$$C = \frac{qkp}{2}$$

swapping success probability (q)

number of users (k)

probability of successfully generating entanglement at link level (p)

need two Bell pairs to generate one end-to-end Bell pair (2)

exact match with CTMC analysis results

Expected number of stored qubits in steady state

When the system is stable,

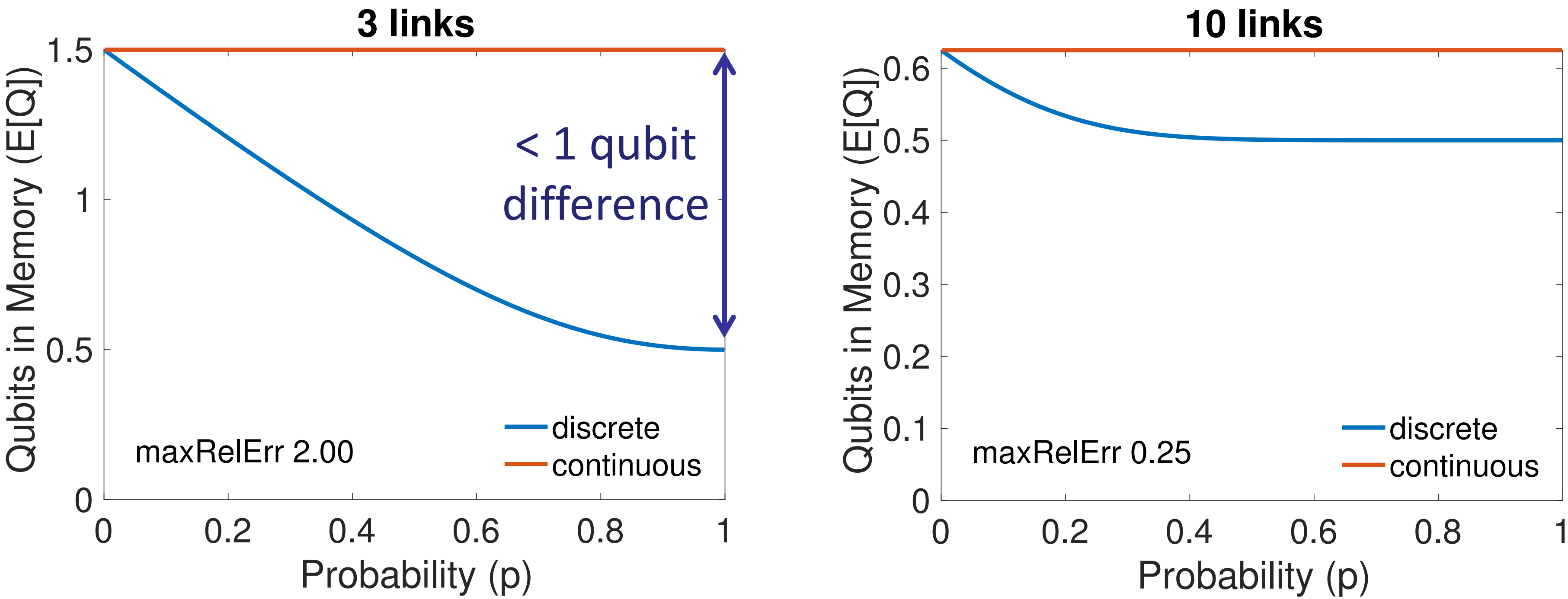
$$E[Q]_{CTMC} = \frac{k}{2(k-2)}$$

$$E[Q]_{DTMC} = \frac{1+\beta}{2(1-\beta)}$$

$$(\beta p + \overline{p})^{k-1}(p + \beta\overline{p}) - \beta = 0, \qquad \beta \in (0,1)$$

$$\overline{p} \equiv 1 - p$$

$E[Q]$ Numerical Comparison



$$\text{maxRelErr}(k) = \max_{p \in (0,1)} \frac{|E[Q]_{DTMC}(k,p) - E[Q]_{CTMC}(k)|}{E[Q]_{DTMC}}$$

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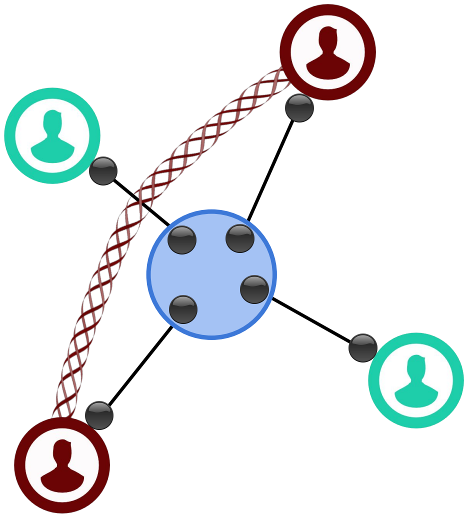


Summary & Future Directions

Summary

- Bipartite entanglement switching problem modeled as a discrete-time Markov chain

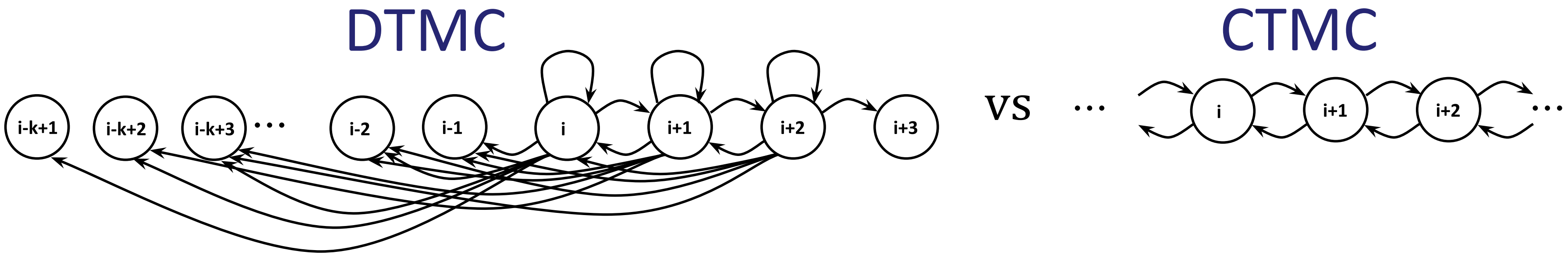
- For a highly idealized switch, not much difference between CTMC and DTMC models

- Nevertheless, problem interesting from queueing-theoretic perspective
  - Kitting process/stochastic assembly-like queue

Future Directions

- Extensions
  - non-identical links
  - non-unit state fidelities
  - finite quantum memories
  - noisy storage/state decoherence

- DTMC becomes infeasible for modeling more complex systems



Thank You!

Questions?

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