Revenue sharing on the Internet: A Case for Going Soft on Neutrality Regulations

ABSTRACT

Revenue sharing contracts between Content Providers (CPs) and Internet Service Providers (ISPs) can act as leverage for enhancing the infrastructure of the Internet. ISPs can be incentivised to make investments in network infrastructure that improve Quality of Service (QoS) for users if attractive contracts are negotiated between them and CPs. The idea here is that part of the revenue of CPs is shared with ISPs to invest in infrastructure improvement. We propose a model in which CPs (leaders) determine contracts, and an ISP (follower) reacts by strategically determining the infrastructure enhancement (effort) for each CP. Two cases are studied: (i) the ISP differentiates between the CPs and puts (potentially) a different level of efforts to improve the QoS of each CP, and (ii) the ISP does not differentiate between CPs and makes equal amount of effort for all the CPs. The last scenario can be viewed as *neutral* behavior by the ISP. Our analysis of optimal contracts shows that preference of CPs for the neutral and non-neutral regime depends on their monetizing power – CPs which can better monetize its demand (stronger CPs) tend to prefer non-neutral regime whereas the weaker CPs tend to prefer the neutral regime. Interestingly, ISP revenue, as well as social utility, are also found to be higher under the non-neutral regime. We then propose an intermediate regulatory regime that we call *"soft-neutral"*, where efforts put by the ISP for all the CPs need not be equal same but the disparity is not wide. We show that the soft-neutral regime alleviates the loss in social utility in the neutral regime and the outcome further improves when CPs determine their contracts through bargaining.

KEYWORDS

Internet economics, revenue sharing, network neutrality

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1 INTRODUCTION

The rapid growth of data-intensive services has resulted in an explosion of the Internet traffic, and it is expected to increase at an even faster rate in the future [9]. To accommodate this increase in traffic, and to provide better Quality of Service (QoS) for end users, Internet service providers (ISPs) need to perform expensive upgradations to their network infrastructure to expand capacity and guarantee low latency. However, with the Internet access fast turning into a low-margin, high-volume commodity item, ISPs are finding it increasingly difficult to recoup their infrastructure investments. Indeed, inter-ISP competition has driven the price for internet access to remain flat over the past several years, even as user expectations in terms of QoS keep growing. On the other hand, the revenues of CPs have seen steady growth, for subscription based as well as advertising supported services. This asymmetry between CP and ISP surplus creates a pressure for surplus transfer from CPs to ISPs; the Netflix-Comcast saga of 2014 (see[7]) being a celebrated example. A justification for such surplus transfer based on cooperative game theory is provided in [17].

CPs may also have the incentive to contribute to ISP capacity expansion, as increased capacity and better QoS trigger higher demand for content and help them earn even higher revenues (mainly from subscriptions and advertisements) [22]. For example, in [13], the authors propose a model in which a Mobile Network Operator leases its edge caches to a CP. This is an increasingly relevant scenario and follows proposals for deploying edge storage resources at mobile 5G networks [2]. Netflix already places a local cache of its most popular content within the data centers of partner ISPs, to enable for seamless, high definition streaming of its videos to its subscribers.[1] Another common mechanism for large CPs like Google and Facebook to (indirectly) subsidize ISP costs is to provide settlement-free points of presence (PoPs) that ISPs can connect to. We are also increasingly witnessing the bundling of certain online services with Internet access; for example, Airtel in India offers complementary Netflix membership and Amazon Prime membership to its postpaid subscribers.

However, while bilateral arrangements between CPs and ISPs of the kind described above (the terms of which are often private) do enable a direct or indirect transfer of surplus from CPs to ISPs, they have been criticized as being violations of the spirit of network neutrality [1]. Network neutrality seeks to make the Internet a level playing field for Internet applications. However, neutrality regulation has thus far been focused narrowly on disallowing ISPs from discriminating between packets corresponding to different applications (in price and/or network priority). Thus, even though bilateral business arrangements between ISPs and CPs that entail local caching, bandwidth expansion at interconnection points, or service bundling, do not legally constitute neutrality violations, they clearly have the effect of skewing the CP marketplace. Indeed, if the content of one partner CP loads faster, or is provided for free to users of an ISP, that would drive consumption away from the competitors of that CP. This makes it harder for smaller CPs, who may not be able to afford such arrangements to grow their user

[1] https://media.netflix.com/en/company-blog/how-netflix-works-with-isps-around-the-globe-to-deliver-a-great-viewing-experience

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base. Thus, activists have argued for stronger network neutrality regulation, that disallows the fragmentation of the Internet into slow and fast lanes, and provides comparable quality of service to all applications and services.

There is thus a tension between incentivizing CPs to contribute towards ISP capacity expansion and network neutrality. Indeed, the primary incentive for CPs to contribute towards Internet infrastructure expansion is to boost their own usage, which gets diluted in the presence of regulation that seeks to level the quality of access for all services.

The goal of this paper is to shed light on the interplay between revenue sharing arrangements between CPs and ISPs, and strong network neutrality regulation. Specifically, we propose a revenue sharing mechanism whereby CPs commit to sharing a portion of their revenues with an ISP, giving the ISP the incentive to make investments that improve the QoS for that CP's content, which in turn would result in higher consumption of that content by users, and higher revenue for the CP. Given that CPs and ISPs act strategically, our aim to shed light on the revenue sharing contracts that would emerge, both in the presence and absence of neutrality regulation that mandates comparable QoS for all CPs.

Specifically, we focus on a monopolistic ISP connecting end users to multiple CPs. Our interest is in determining revenue sharing arrangements/contracts between each CP and the ISP, given that all agents act strategically. We distinguish two cases for the effort (investment) made by the ISP. In one case, we allow the ISP to make different amounts of effort for each CP, and in the other case, the ISP is constrained to make an equal amount of effort for all CPs. The former case corresponds to a 'non-neutral' regime, where the ISP is allowed to provide differentiated levels of QoS to different CPs, and the latter case corresponds to an idealized 'neutral' regime where the ISP cannot differentiate between CPs. We compare the utility of each player and the social utility under both the regimes and analyze which regime is preferred by each player.

We observe that the ISP always prefers the non-neutral regime, whereas the preference of CPs depends on their monetization power; stronger CPs that have better monetization power prefer the non-neutral regime while weaker CPs prefer the neutral regime. However, irrespective of the CPs monetization power, the social utility is always higher in the non-neutral regime. Though the neutral regime is considered to be ideal for the long term health of the Internet ecosystem, that it diminishes social utility (in the short term) is a cause of concern. This means policy makers need to act cautiously when attempting to introduce strong network neutrality regulation.

To mitigate the drawbacks of both the neutral and the non-neutral regimes, we propose a softer, intermediate approach to neutrality, where the ISP is allowed to put provide differentiated levels of QoS to different CPs, but only to an extent. Mathematically, under the proposed soft-neutral regime, the minimum effort the ISP makes for a CP cannot be lower than a prescribed fraction of maximum effort it makes among all CPs. Another complementary regulatory intervention we propose, inspired by [10], is to implement a transparent platform for CPs to 'bargain' on their revenue sharing contracts, to avoid the *tragedy of the commons* that results from non-cooperative interaction. Remarkably, we show these interventions, if well calibrated, can provide an even higher social utility than the neutral and non-neutral regimes.

Our contributions and observations are as follows:

- We model revenue sharing contracts between CPs with an ISP. The CPs first offer a contract to the ISP, and ISP accordingly determines the effort for each CP seeking to maximize its own utility. This results in a Stackelberg game with multiple leaders (CPs) and single follower (ISP).
- We analyze the equilibrium contracts in this leader-follower interaction in two regimes: one where the ISP can make a different, customized level of effort for each CP (non-neutral), and one where it is constrained to make equal efforts for all CPs (neutral).
- When the CPs are symmetric with respect to their monetization capabilities, we show that all the players prefer the non-neutral regime as their utilities are higher.
- When the CPs are asymmetric with respect to their monetization capability, we show that the CPs that can better monetize the demand for their traffic prefer the non-neutral regime, whereas CPs with weaker monetization power may prefer either regime depending on their relative monetization power.
- The ISP always prefers the non-neutral regime. Moreover, social utility (defined as the sum of the utilities of all players) is also higher in the non-neutral regime.
- We propose an intermediate *soft-neutral* regime, which, in conjunction with a bargaining platform for CPs, alleviates the loss in social utility in the neutral regime, while also limiting imbalances in QoS for content from different CPs.

This paper is organized as follows. In Section 2 we discuss the problem setup and define contracts under the neutral and non-neutral regime. We study the equilibrium contracts under the neutral and non-neutral regimes for the symmetric CP case in Section 3 and for the asymmetric CP case in Section 4. The soft-neutral regime is discussed in Section5. Conclusions and future extensions are discussed in Section 6. Proofs of all stated results can be found in the appendix.

1.1 Related literature

Several works [8, 10–12, 19, 20] study the possibility of content charges by ISPs to recover investment costs. In [10], a purely neutral scenario, where CPs bargain in order to determine how much of their revenue contribute towards ISP capacity expansion, is considered. In [11] and [12], the authors investigate the feasibility of ISPs charging a content charge to CPs, and evaluate its effect by modeling the Stackelberg game between CPs and ISPs. In [20] and [19], a revenue-sharing scheme is proposed when the ISP provides a content piracy monitoring service to CPs for increasing the demand for their content. This work is extended to two ISPs competing with each other in [8] where only one of them provides the content piracy monitoring service.

Several studies considered cooperative settlement between service providers for profit sharing [6, 15, 16] where the mechanisms are derived using the Shapely value concept. In [15], authors investigated a profit settlement problem between content and eyeball

ISPs. Authors extended this work in [16] for three classes of ISPs: content, transit, eyeball, and provided generalizations of all results in [15] along with a closed-form Shapely solution. In [6], authors considered the revenue sharing problem between ISP and CP, where ISP deploys a cache for a CP, and proposed Shapely value concept of coalition games for the splitting of the profit.

Few studies also considered premium peering agreement between ISP and CPs, in which ISPs offers better connectivity at higher peering prices [4, 5, 14, 23]. In [5], authors used Stackelberg game with the ISP being leader and CPs being followers study peering prices. [4] used Nash Bargaining solution concept to obtain premium peering prices. [14] used a choice model to determine CP's peering decision based on the value of direct peering. [23] studied the problem where CP determines additional capacity and cache to purchase from ISP, given ISP's premium prices.

However, none of the above papers analyse the crucial interplay between revenue sharing between CPs and ISPs on the internet, and network neutrality.

2 PROBLEM DESCRIPTION

We consider multiple Content Providers (CPs) and a single Internet Service Provider (ISP) that connects end users to the content of the CPs. Each CP can enter into a revenue sharing contract with the ISP, whereby the ISP commits to infrastructure improvements to boost demand for the CP's content, and in return, the CP agrees to share part of the resultant increase in its revenue with the ISP. One example of infrastructure investment by the ISP that helps boost CP demand is caching—the ISP may cache part of a CP's content locally, improving the QoS seen by users, resulting in increased consumption of that CPs content, and greater revenue for the CP from subcriptions/advertisements.

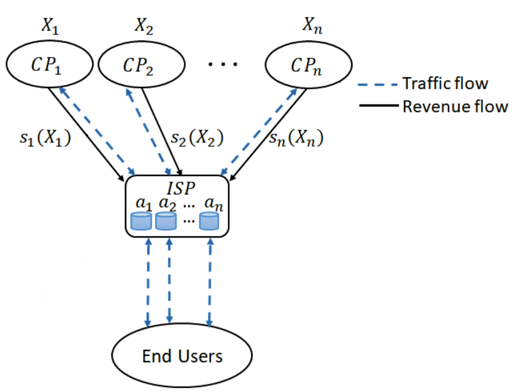


Figure 1: Revenue flow between CPs, ISP and End Users.

Let n denote the number of CPs and $\mathcal{N} = \{1, 2, \ldots, n\}$ denote the set of CPs. For each $i \in \mathcal{N}$ we denote i-th CP as CP_i. The amount of effort/investment made by the ISP to improve demand for the content of CP_i is quantified by a positive number $a_i \in \mathcal{R}_+$. The resulting increase in the revenue of CP_i over a pre-specified horizon (say a billing cycle) is denoted by $X_i \in \mathcal{R}_+$. In return, CP_i shares part of its revenue increase with the ISP to incentivize its investment; this share is determined by the sharing function $s_i : \mathcal{R}_+ \rightarrow \mathcal{R}_+$, which is also referred to as the contract/agreement. Specifically, the CP_i makes a payment of $s_i(X_i)$ to the ISP as part of this contract. Thus, the effective revenue increase of CP_i is $X_i - s_i(X_i)$. Our goal in this work is to determine the revenue sharing contracts that would emerge between rational CPs and the ISP, and to shed light on the implications of these contracts. The interactions between the agents of our model are depicted in Figure 1. The details of the model are presented below. Since we are interested in the increments of demand and revenue on the CP side due to the additional infrastructure investments by the ISP, we treat the baseline (corresponding to zero ISP effort) demand/revenue to be zero without loss of generality.

Demand & CP revenue

The demand (increment) D_i for the content of CP_i depends on the effort a_i made by the ISP. It is natural to model D_i as an inceasing concave function of a_i, consistent with the law of diminishing returns. For analytical tractability, we assume that D_i grows logarithmically in a_i and is given by $D_i := \log(a_i + 1), \forall i \in \mathcal{N}$.

The resulting revenue (increment) for CP_i is assumed to be proportional to the demand (increment) and given by $X_i = r_i D_i$, where r_i is a constant that captures how each additional unit of demand translates to earnings. For example, r_i could be revenue per click for CP_i when D_i is interpreted as total number of clicks on CP_i's content. Thus, the revenue generated by CP_i is then $X_i = r_i \log(a_i+1)$.[2]

Utilities and Objective

We restrict attention to linear revenue sharing contracts between the ISP and CPs. Specifically, these contracts are of the form $s_i(X_i) = \beta_i X_i$, where $\beta_i \in [0, 1]$ for all $i \in \mathcal{N}$. The contracts, paremterized by β_i, $i \in \mathcal{N}$, are offered by the (strategic) CPs to the ISP, who then (strategically) determines its efforts towards infrastructure improvements. The cost incurred by the ISP is assumed to be proportional to its effort, i.e., ISP cost is given by $c \sum_i a_i$, where c is a positive constant.

Thus, the utility of CP_i in this model is given by:

$$U_{\text{CP}_i} = (1 - \beta_i) r_i \log(a_i + 1), \tag{1}$$

and the utility of the ISP is:

$$U_{\text{ISP}} = \sum_i (\beta_i r_i \log(a_i + 1) - c a_i) \tag{2}$$

Neutral vs Non-Neutral regime

We distinguish two scenarios based on the differentiation in the efforts by the ISP for each CP. We say that the network is *neutral* if ISP is constrained by regulation to make the same effort to improve the QoS of all CPs (irrespective of diversity in the revenue-shares it can get from them), i.e.,

$$a_1 = a_2 \ldots = a_n =: a. \tag{3}$$

We say that the network is *non-neutral* if there is no such constraint on the ISP, i.e., $a_1 \neq a_2 \ldots \neq a_n$ is permitted. In the non-neutral regime (which is essentially the status quo at present), the ISP is free to focus its efforts on improving the QoS of 'bigger' CPs that can better monetize their content, and can therefore also offer more favourable contracts to the ISP.

Stackelberg game formulation

[2] Our model generalizes trivially to the case where $D_i := d_i \log(a_i + 1)$, i.e., the demand scales with a CP-specific multiplicative constant; this constant can simply be absorbed into r_i.

We model the interactions between the CPs and the ISP as follows. We consider a leader-follower interaction, with the CPs acting as leaders and the ISP as the follower. The CPs lead by announcing the revenue sharing contracts, i.e., CP_i offers the contract $\beta_i \in [0, 1]$ to the ISP. The ISP then responds to these contracts to determine its efforts $(a_i,\ i \in \mathcal{N})$. Note that under the neutral regime, the ISP's response is constrained to satisfy (3).

We now point out how the above interaction model plays out differently in the neutral and the non-neutral regime.

Neutral regime: In the neutral regime, the best response of the ISP (the follower) is to optimize its utility (2) subject to the constraint (3). This best response is easily seen to be:

$$a^N(\beta) = \max\left(\frac{\sum_i \beta_i r_i}{nc} - 1, 0\right). \tag{4}$$

It is easy to see that under this response, the utility of each CP depends on the actions of all CPs. Thus, we model the emerging revenue sharing contracts as a Nash equilibrium between the CPs. Specifically, the utility of each CP is given by

$$U^N_{\text{CP}_i}(\beta) = (1-\beta_i) r_i \log(a^N(\beta) + 1),$$

and a Nash equilibrium β^N between the CPs satisfies, for all $i \in \mathcal{N}$,

$$U^N_{\text{CP}_i}(\beta_i^N, \beta_{-i}^N) \geq U^N_{\text{CP}_i}(\beta, \beta_{-i}^N) \quad \forall \quad \beta \in [0, 1].$$

Non-neutral regime: In the non-neutral regime, the interactions between each CP and the ISP get decoupled. This is because the best response of the ISP is to make an effort for each CP that depends only on the contract offered by that CP. Thus, the optimal effort by the ISP in response to the contract offered by CP_i is

$$a_i^{NN}(\beta_i) = \max\left(\frac{\beta_i r_i}{c} - 1, 0\right).$$

Given this, the optimal strategy for CP_i is to offer the contract that maximizes its utility, i.e., the solution to

$$\max_{\beta_i \in [0,1]} \quad (1-\beta_i)\log(a_i^{NN}(\beta_i) + 1) r_i.$$

This concludes our model description. In the following sections, we analyse the implications of the above interaction models on the payoffs of all agents.

3 SYMMETRIC CPS

In this section we consider the symmetric case where revenue per unit demand for all the CPs is the same, i.e., $r_1 = r_2, \ldots, = r_n := r$. In other words, the CPs are symmetric with regards to the ability to monetize their content. In this setting, we analyze the equilibrium contracts arising in the neutral as well as non-neutral regime, and the resulting surplus of the CPs and the ISP. Our results highlight, surprisingly, that even when the CPs are symmetric, the imposition of neutrality actually shrinks the surplus of all parties involved. Moreover, this 'loss of surplus' becomes more pronounced as the number of CPs grows.

3.1 Non-neutral regime

As mentioned before, in the non-neutral regime, the interactions between CPs and the ISP get decoupled. Under our symmetry assumption, this means that each CP would offer a contract that solves:

$$\max_{\beta \in [0,1]} \quad (1-\beta) r \log\left(\max\left(\frac{\beta r}{c}, 1\right)\right).$$

Moreover, we note that it is only interesting to consider the case $r > c$. Indeed, since the monetization resulting from ISP effort a_i for CP_i equals $r\log(1+a_i)$, the marginal monetization is at most r. Thus, if $r \leq c$, which is the marginal cost associated with infrastructure expansion, it is not worthwhile for CPs to make investments to grow the demand. Henceforth we assume that $r > c$.

The following result characterizes the equilibrium contracts between each CP and the ISP. The contracts are expressed in terms of the LambertW function computed on its principle branch, denoted as $W(\cdot)$ (see [3]).

Theorem 1. *The equilibrium contract between each CP and the ISP is given by*

$$\beta^{NN} := \beta_i^{NN} = \frac{1}{W\left(\frac{r}{c}e\right)} \text{ for all in} \in \mathcal{N}. \tag{5}$$

Since $W(\cdot)$ in strictly increasing and $W(e) = 1$, it follows that $\beta^{NN} \in (0, 1)$ when $r/c > 1$. Moreover, note that equilibrium fraction β^{NN} of CP revenue that is shared with the ISP is a strictly decreasing function of the ratio r/c, as might be expected.

Using Theorem 1, one can characterize the equilibrium effort of the ISP as well as the surplus of each agent.

Corollary 1. *The equilibrium effort by the ISP for each CP is given by*

$$a^{NN} := a_i^{NN} = \frac{r\beta^{NN}}{c} - 1 > 0. \tag{6}$$

The equilibrium surplus of CP_i, $i \in \mathcal{N}$ is given by

$$U^{NN}_{CP_i} = (1-\beta^{NN}) r \log(a^{NN} + 1) = \frac{(1-\beta^{NN})^2}{\beta^{NN}} r > 0. \tag{7}$$

Finally, the equilibrium surplus of the ISP is given by

$$U^{NN}_{ISP} = nr + nc - 2n\beta^{NN} r > 0. \tag{8}$$

Note that so long as $r > c$, the equilibrium contracts award each CP and the ISP a positive surplus.

3.2 Neutral regime

We now consider the neutral regime. The CPs are still assumed to be symmetric, only the ISP is now *constrained* to make the same investment decision for all CPs, i.e., $a_1 = \ldots = a_n := a$. The surplus of CP_i in this case, after substituting the optimal ISP effort (4) simplifies to:

$$(1-\beta_i) r \log\left(\max\left(\frac{\sum_{j=1}^n \beta_j r}{nc}, 1\right)\right).$$

Since the surplus of each CP in the neutral regime depends on the actions of all CPs, we seek contract profiles $(\beta_i^N, i \in N)$ that constitute a *Nash equilibrium* between CPs. These equilibria are characterized completely in the following theorem. As before, the only scenario of interest is $r > c$.

Theorem 2. *In the neutral regime with $1 < r/c \leq n$, there are exactly two Nash equilibrum profiles $(\beta_i^N, i \in \mathcal{N})$, both symmetric:*

$$\beta_i^N = \beta^N := 0. \tag{9}$$

and

$$\beta_i^N = \beta^N := \frac{1}{nW\left(\frac{r}{nc}e^{1/n}\right)}. \quad (10)$$

When $r/c > n$, only one Nash equilibrium profile $(\beta_i^N, i \in \mathcal{N})$ exists and is given by (10).

Note that when $1 < r/c \leq n$, unlike in the non-neutral regime, making no contributions to the ISP, resulting in zero suplus for all parties, is an equilibrium between the CPs. The other equilibrium, given by (10), results in a positive surplus for all parties (as is shown in the following corollary). In the remainder of this section, we will refer to this latter equilibrium as the *non-zero* equilibrium.

Corollary 2. *Consider the neutral regime with $r > c$. Under the non-zero equilibrium:*

- *The effort put by the ISP for each CP_i, $i \in N$ is given by*

$$a^N := a_i^N(n) = \max\left(\frac{\beta^N r}{c} - 1, 0\right). \quad (11)$$

- *The surplus of CP_i, $i \in N$ is given by*

$$U_{CP_i}^N = (1-\beta^N) r \log(a^N + 1) = \frac{\left(1-\beta^N\right)^2}{n\beta^N} r > 0. \quad (12)$$

- *The surplus of ISP is given by*

$$U_{ISP}^N = r + nc - (n+1)\beta^N r > 0. \quad (13)$$

3.3 Neutral regime v/s Non-neutral regime

Having now characterized the equilibrium contracts and the surplus of each CP and the ISP under the neutral and the non-neutral regime, we are now in a position to compare the two regimes. As the following result shows, the non-neutral regime is actually better *for all parties* as compared to the neutral regime.

Theorem 3. *Suppose $r > c$, and $n \geq 2$. In the symmetric case, at equilibrium, the following statements hold in the non-neutral regime.*

1. *CPs share a higher fraction of their revenue with the ISP, i.e., $\beta^{NN} > \beta^N$.*
2. *The effort by the ISP for each CP is higher, i.e., $a^{NN} > a^N$*
3. *The surplus of each CP is higher, i.e., $U_{CP_i}^{NN} > U_{CP_i}^N$ for all $i \in N$*
4. *The surplus of the ISP is higher, i.e., $U_{ISP}^{NN} > U_{ISP}^N$.*

The above result highlights that, surprisingly, constraining the ISP to be neutral is actually sub-optimal for all parties, even when the CPs are symmetric. In other words, the non-neutral regime is actually preferable to the ISP as well as the CPs. Intuitively, the reason for this *tragedy of the commons* is that the imposition of neutrality skews the payoff landscape for each CP, such that the 'benefit' of any additional investment it makes gets 'shared' across all CPs. This induces the CPs to commit smaller fractions of their revenues to the ISP, which in turn results in a lower ISP effort, and a lower demand growth for all CPs. Indeed, as we show below, this effect gets further magnified with an increase in the number of CPs.

3.4 The effect of number of CPs

In the non-neutral regime, the interactions between the different CPs and the ISP are decoupled, implying that the impact of scaling n is trivial. Thus, we now study the impact of scaling n in the neutral regime on the equilibrium ISP effort, and the surplus of each agent. Note that when $n = 1$, the neutral and the non-neutral regime coincide. Our main result is the following.

Theorem 4. *Suppose that $r > c$. In the neutral regime, the non-zero equilibrium satisfies the following properties.*

1. *β^N is a strictly decreasing function of n.*
2. *The effort by the ISP for each CP (a^N) is a strictly decreasing function of n, even though the total effort (na^N) by the ISP is a strictly increasing function of n.*
3. *The surplus of each CP is a strictly decreasing function of n, and $\lim_{n\to\infty} U_{CP_i}^N(n) = 0$.*
4. *The surplus of the ISP is eventually strictly decreasing in n, and $\lim_{n\to\infty} U_{ISP}^N(n) = 0$.*

Theorem 4 highlights that an increase in the number of CPs further exacerbates the sub-optimality of the neutral regime for the CPs as well as the ISP. As before, the explanation for this is that with increasing n, the surplus resulting from an additional contribution by any *CP* gets 'split' further, thus disincentivising the CPs from offering a significant fraction of their revenues to the ISP. The variation of ISP utility as a function of n is depicted in Figure 2 for different values of r/c. In all cases, the utility increases with n for small n, but eventually diminishes with increasing n, consistent with Theorem 4.

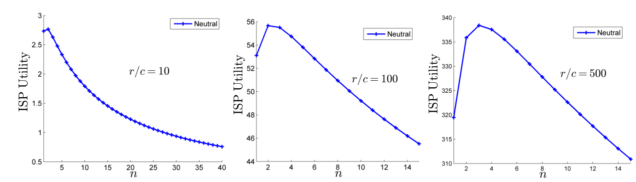


Figure 2: ISP utility in the neutral regime as n varies for different r/c.

4 ASYMMETRIC CPS

In this section we study the asymmetric case where the monetizing power of the CPs need not be the same, i.e., $r_i \neq r_j$ for $i \neq j$. Our goal is to highlight how disparity in CP monetizing power influences the actions and utilities of different agents. To simplify the presentation, we focus on the case with two CPs ($n = 2$) and without loss of generality assume that monetization power of CP_1 exceeds that of CP_2, i.e., $r_1 > r_2$. We refer to CP_1 as the *dominant* CP, and CP_2 as the *non-dominant* CP.

As discussed in Section 3, the case $r_i/c \leq 1$ for all $i \in \mathcal{N}$ is not interesting as none of the CPs would have the incentive to contribute towards infrastructure investment by the ISP. Thus, in this section, we restrict ourselves to the case where $r_1/c > 1$. The following results characterize the equilibrium contracts for the neutral and the non-neutral regime.[3]

[3] The characterization of equilibrium contracts can actually be done for any n; see Appendix 12.

4.1 Equilibrium contracts

In the non-neutral regime, the interactions between each CP and the ISP remain decoupled, and thus the equilibrium contracts follow easily from Theorem 1.

COROLLARY 3. *In the non-neutral regime, the equilibrium contract* $(\beta_1^{NN}, \beta_2^{NN})$ *is as follows:*

$$\beta_i^{NN} = \begin{cases} 0 & \text{if } \frac{r_i}{c} \le 1, \\ \frac{1}{W(\frac{r_i}{c}e)} & \text{if } \frac{r_i}{c} > 1. \end{cases}$$

Note that when $\frac{r_i}{c} \le 1$, the equilibrium contract between CP$_i$ and the ISP is not uniquely defined, since any $\beta_i \in [0, 1]$ would result in zero surplus to CP$_i$.

Next, we characterize equilibrium contract in the neutral regime.

THEOREM 5. *Consider the neutral regime, with* $r_1 > r_2$. *If* $r_1/c \le 2$ *then* $(\beta_1^N, \beta_2^N) = (0, 0)$. *If* $r_1/c > 2$, *then the equilibrium contract is given by:*

$$(\beta_1^N, \beta_2^N) = \begin{cases} (\overline{\beta}_1, \overline{\beta}_2) & \text{if } \frac{r_1+r_2}{r_1-r_2} > 2W\left(\frac{r_1+r_2}{4c}\sqrt{e}\right), \\ \left(\frac{1}{W(\frac{r_1}{2c}e)}, 0\right) & \text{otherwise,} \end{cases} \tag{14}$$

where

$$\overline{\beta}_1 = \frac{r_1 + r_2}{4r_1 W\left(\frac{r_1+r_2}{4c}\sqrt{e}\right)} - \frac{r_2 - r_1}{2r_1}, \overline{\beta}_2 = \frac{r_1 + r_2}{4r_2 W\left(\frac{r_1+r_2}{4c}\sqrt{e}\right)} - \frac{r_1 - r_2}{2r_2}.$$

When $r_1/c \le 2$, the equilibrium contract is not unique, though the outcome is that ISP effort equals zero. When $r_1/c > 2$, the equilibrium contract is unique, and at least one CP (specifically, CP$_1$) is guaranteed to contribute a positive fraction of her revenue to the ISP. Note that when $r_1/c \in (1, 2]$, there is no CP contribution in the neutral regime, even though there is in the non-neutral regime.

To interpret the equilibrium when $r_1/c > 2$, let $r_1^* := r_1^*(r_2)$ denote the value of r_1 that satisfies the following relation for a given r_2

$$\frac{r_1 + r_2}{r_1 - r_2} = 2W\left(\frac{r_1 + r_2}{4c}\sqrt{e}\right).$$

For $r_1 \le r_1^*$ the condition in (14) holds where revenue shared by both the CPs is strictly positive, i.e., $\beta_i^N > 0$ for all $i \in \mathcal{N}$. For $r_1 > r_1^*$ the condition in (14) fails in which only CP$_1$'s share is strictly positive and CP$_2$ does not share anything, i.e., $\beta_1^N > 0$ and $\beta_2^N = 0$. Further, it is easy to verify that r_1^* is monotonically increasing in r_2 and $r_1^* > r_2$.

4.2 Comparison between Neutral and Non-neutral regimes

Having characterized the equilibrium contracts in both regimes, we compare and contrast the neutral and non-neutral regimes in the remainder of this section. We begin by comparing the equilibrium contracts, followed by CP/ISP utility, social utility, and finally ISP effort.

4.2.1 ***Contracts***. The following proposition provides a comparison of the equilibrium contracts in both the regimes.

PROPOSITION 1. *Fix* $r_2 > 0$. *We have*

- *For* $r_1 > r_2$, $\beta_2^{NN} \ge \beta_2^N$. *Moreover,* β_2^N *decreases in* r_1.
- *For* $r_1 \ge r_1^*$, $\beta_1^N > \beta_1^{NN}$. *Moreover,* β_1^N *decreases in* r_1 *for* $r_1 \ge r_1^*$.

The conclusions of Proposition 1 are summarised in the scatter plot in Fig. 3a. Note that the non-dominant CP always contributes a smaller fraction of its revenue in the neutral regime. With the dominant CP, the contribution factor is larger in the neutral regime when the revenue rates are highly asymmetric (see the green region in Figure 3a, and larger in the non-neutral regime when the revenue rates are symmetric (see the red region in Figure 3a). A sufficient condition for the former is $r_1 \ge r_1^*(r_2)$. The latter observation is of course consistent with Theorem 3, which dealt with the case of perfect symmetry.

Proposition 1 also establishes monotonicity properties of the sharing contracts of CP$_1$ in the neutral regime in r_1 for a fixed r_2. While β_2^N decreases in r_1, β_1^N eventually decreasing in r_1; see Figs. 3b and 3c. Note that β_1^N can actually be increasing with respect to r_1 when the revenue rates are nearly symmetric, in contrast with the non-neutral setting.

4.2.2 ***Utility of CPs***. The following proposition characterizes preference of the CPs for the neutral and non-neutral regime.

PROPOSITION 2. *Fix an* r_2. *We have*

- *For all* $r_1 > r_2$, *CP*$_1$ *prefers the non-neutral regime.*
- *For all* $r_1 \ge r_1^*$, *CP*$_2$ *prefers the neutral regime.*

Figure 4a is a scatter plot that compares the utilities of the different players across both regimes. Note that the dominant CP has higher utility in the non-neutral regime as can be observed from the red and magenta regions. This is because in the neutral regime, the dominant CP is 'forced' to pay for capacity investments that also benefit the non-dominant CP. Indeed, note that in the region $r_1 \ge r_1^*$, the dominant CP shares a smaller fraction of its revenue in the non-neutral regime, but still ends up with a higher utility. Interestingly, the non-dominant CP obtains a higher utility in the neutral regime when the revenue rates are highly asymmetric (see the pink region in Fig. 4a) A sufficient condition for this is $r_1 \ge r_1^*$. This is of course due to the 'subsidization' it receives from the dominant CP. On the other hand, when the revenue rates are nearly symmetric, even the non-dominant CP prefers the non-neutral regime, once again consistent with Theorem 3. The above observations are further illustrated in Figs. 4b and 4c.

4.2.3 ***ISP Utility***. We next compare the utility of the ISP in the non-neutral and neutral regime. Its value in the non-neutral regime is given by:

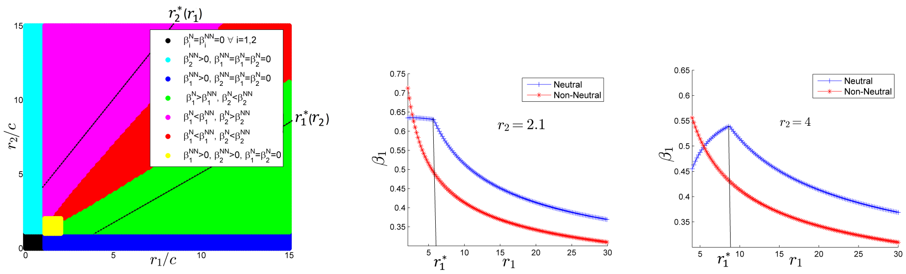
$$U_{\text{ISP}}^{NN} = (1 - 2\beta_1^{NN})r_1 + (1 - 2\beta_2^{NN})r_2 + 2c,$$

and in the neutral regime for all $r_1 \ge r_1^*$ is given by:

$$U_{\text{ISP}}^{N} = (1 - 2\beta_1^{N})r_1 + 2c.$$

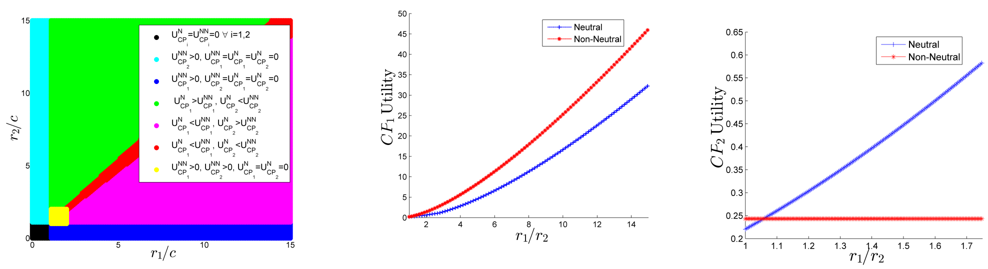
The utility for $r_1 < r_1^*$ in the neutral regime is cumbersome and we skip its expression. The following lemma demonstrates the ISPs earnings are higher in the non-neutral regime when monetization power of the dominant CP is much larger than the other, i.e., r_1 is much larger than r_2.

LEMMA 1. *There exists* $r_1^b > r_1^*$, *such that for all* $r_1 > r_1^b$ *the ISP's utility is higher in the non-neutral regime.*



(a) (b) (c)

Figure 3: Fig. 3a gives scatter-plot of βs. Figs. 3b and 3c shows variation of equilibrium β_1 vs r_1 under neutral and non-neutral regime.



(a) (b) (c)

Figure 4: Fig. 4a shows scatterplot for the CP utilities at equilibrium. Figs. (4b) & (4c) compare CP$_1$ utility in both regimes as r_1 varies.

A general comparison of ISP utility in the two regimes is not analytically tractable. We give a numerical illustration in Figure 5. As seen in the first figure, utility of ISP in the non-neutral regime is higher than in the neutral regime for all r_1 for a given r_2 and c. The scatter plot in the second figure shows that this observation extends over the entire parameter range, i.e., ISP utility is higher in the non-neutral regime.

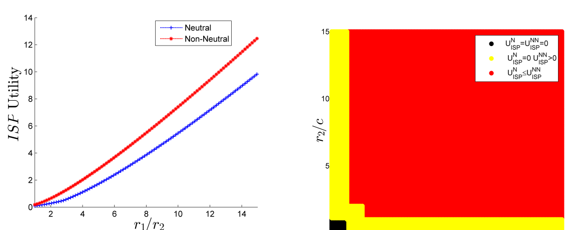


Figure 5: The first figure compares ISP utility in neutral and non-neutral with $c = 1 r_2 = 2$. The second figure gives a scatter plot.

4.2.4 ***Social Utility***.

The social utility in the non-neutral and neutral regimes are given, respectively, as follows:

$$SU^{NN} = U^{NN}_{CP_1} + U^{NN}_{CP_1} + U^{NN}_{ISP}$$

$$= r_1 \log\left(\frac{\beta_1^{NN} r_1}{c}\right) + r_2 \log\left(\frac{\beta_2^{NN} r_2}{c}\right) - (\beta_1^{NN} r_1 + \beta_2^{NN} r_2) + 2c,$$

$$SU^{N} = U^{N}_{CP_1} + U^{N}_{CP_1} + U^{N}_{ISP}$$

$$= (r_1 + r_2) \log\left(\frac{\beta_1^{N} r_1 + \beta_2^{N} r_2}{c}\right) - (\beta_1^{N} r_1 + \beta_2^{N} r_2) + 2c.$$

As it is not easy to compare the social utilities analytically, we resort to numerical comparison of the utilities in Figure 6. As seen, social utility in the non-neutral regime dominates that in the neutral regime for all values of r_1 for a given r_2 and c. The scatter plot in the

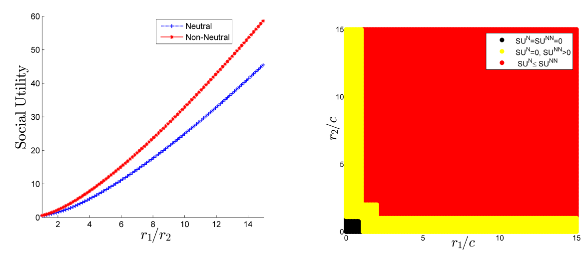


Figure 6: Comparison of social utility between neutral and non-neutral regime for $c = 1, r_2 = 2$ and scatter plot.

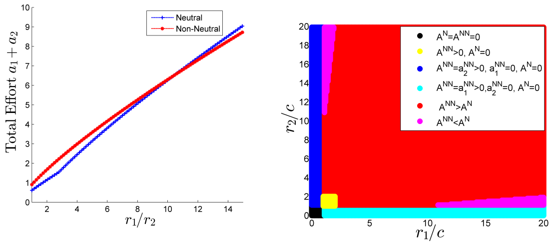


Figure 7: Scatter plot for comparison between total effort (investment) by ISP in non-neutral and neutral regime over different range of r_i/c

second part of the figure shows that the observation holds for all parameter values—social utility is higher in the non-neutral regime.

4.2.5 ***Total Effort by ISP.*** Finally we compare the total effort by ISP for CPs in the non-neutral and neutral regime given, respectively, as follows

$$A^{NN} = a_1^{NN} + a_2^{NN} = \begin{cases} \dfrac{\frac{r_1}{c}}{W(\frac{r_1}{c}e)} + \dfrac{\frac{r_2}{c}}{W(\frac{r_2}{c}e)} - 2 & (\text{for } r_1, r_2 > c) \\ \dfrac{\frac{r_1}{c}}{W(\frac{r_1}{c}e)} - 1 & (\text{for } r_1 > c,\ r_2 \le c) \end{cases},$$

$$A^N = 2a^N = 2\left(\frac{\beta_1^N r_1}{2c} - 1\right) = \frac{\frac{r_1}{c}}{W(\frac{r_1}{2c}e)} - 2 \quad (\text{ for } r_1 > r_1^*).$$

As before, for the neutral regime, we omit the expression for total ISP effort when $r_1 \le r_1^*$. The following lemma compares the ISP effort across the neutral and non-neutral regimes.

Lemma 2. *There exists a threshold $r_1^a > r_1^*$ such that when $r_1 > r_1^a$, the total effort by ISP is higher in neutral regime than in the non-neutral. The threshold satisfies:*

$$r_1^a\left(\frac{1}{W(\frac{r_1^a}{2c}e)} - \frac{1}{W(\frac{r_1^a}{c}e)}\right) = \frac{r_2}{W(\frac{r_2}{c}e)} \quad (\mathit{for}\ r_2 > c),$$

$$\frac{r_1^a}{c}\left(\frac{1}{W(\frac{r_1^a}{2c}e)} - \frac{1}{W(\frac{r_1^a}{c}e)}\right) = 1 \quad (\mathit{for}\ r_2 \le c).$$

It can be seen from above equation that r_1^a is monotonically increasing in r_2. Interestingly, the above lemma implies that in the presence of extreme asymmetry in the CP revenue rates, the neutral regime produces a higher ISP effort, while also resulting in a lower social utility compared to the non-neutral regime. Intuitively, this is because the neutral regime forces the ISP to match its efforts across both CPs, diminishing the effort directed at increasing the consumption of the (better monetizable) content of the dominant CP.

To summarize the key take-aways from this section, suppose that $r_1 \gg r_2$; this corresponds to extreme asymmetry in the monetization capabilities of the CPs, and provides the most contrast with the symmetric setting considered in Section 3. When $r_1 \gg r_2$, the social utility, ISP utility, as well as the utility of the dominant CP are higher under the non-neutral regime. On the other hand, the non-dominant CP is better off in the neutral regime, being able to free-ride on the contributions made by the dominant CP.

These results, in conjunction with those in Section 3, suggest that regulators must exercise caution when considering strong network neutrality regulation seeking to match Internet access quality across different online services.

5 SOFT NETWORK NEUTRALITY

In previous sections, we shed light on the drawbacks of enforcing strong network neutrality vis-à-vis revenue sharing between CPs and ISPs on the Internet. The neutral regime produces lower social utility compared to the non-neutral regime, and can also be worse for all parties involved. There are broadly two reasons for the inferior outcomes we observe under the neutral regime.

1. *Free-riding*: In the presence of extreme asymmetry between the monetization capabilities of CPs, the neutral regime allows the non-dominant CP to free-ride on the contributions of the dominant CP. This discourages contributions from the dominant CP, and also creates less value in the ecosystem.
2. *Tragedy of the commons:* The non-cooperative framework sometimes results in equilibria that are worse for all players as compared to alternative (non-equilibrium) configurations. The game theoretic remedy for this is to allow signalling between the agents, allowing them to 'bargain' and enter into binding agreements to operate at a mutually beneficial configuration. Capturing this type of interaction is the goal of bargaining theory [18].

In this section, we propose two regulatory interventions to address the two issues highlighted above. The first is to 'soften' the requirement of strong neutrality, and the second is to operate a transparent bargaining platform for CPs to jointly determine their revenue sharing contracts for the ISP.

5.1 Soft neutrality

The idea of soft-neutrality is that is that the ISP is allowed to differentiate the QoS provided for the content of different CPs to a limited extent. Specifically, the regulator specifies a threshold $\rho \in (0, 1)$ such that the ISP is constrained to satisfy

$$\min_{1 \le i \le n} (a_i) \ge \rho \max_{1 \le i \le n} (a_i). \tag{15}$$

Note that one corner case, $\rho = 0$, corresponds to the non-neutral regime, and the other, $\rho = 1$, corresponds to the neutral regime.

The best response of the ISP, to a vector of contracts β from the CPs, subject to the soft-neutrality constraint (15) is described in the following lemma. For simplicity, we restrict attention to the case $n = 2$ throughout the rest of this section.

Lemma 3. *Suppose $n = 2$. The optimal response of the ISP to the CP contracts β_1, β_2 is as follows:*

- *If $a_i^{NN} = \frac{\beta_i r_i}{c} - 1; i = 1, 2$ satisfies soft-neutrality constraint, then*
$$a_i^{SN} = a_i^{NN} = \frac{\beta_i r_i}{c} - 1; i = 1, 2$$
- *If $a_1^{NN} < \rho a_2^{NN}, a_2^{NN} \ge \rho a_1^{NN} \& (\beta_2 r_2 + \beta_1 r_1 \rho) - c(\rho + 1) > 0$, then*
$$a_1^{SN} = \rho a_2^{SN}, a_2^{SN} = \frac{-B + \sqrt{B^2 - 4AC}}{2A}$$

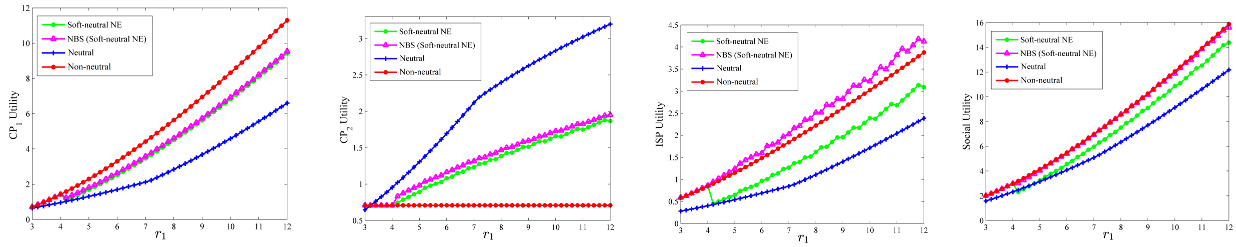


Figure 8: We set $\rho = 0.3$, $c = 1$, $r_2 = 3$ and vary r_1 from 3 to 12.

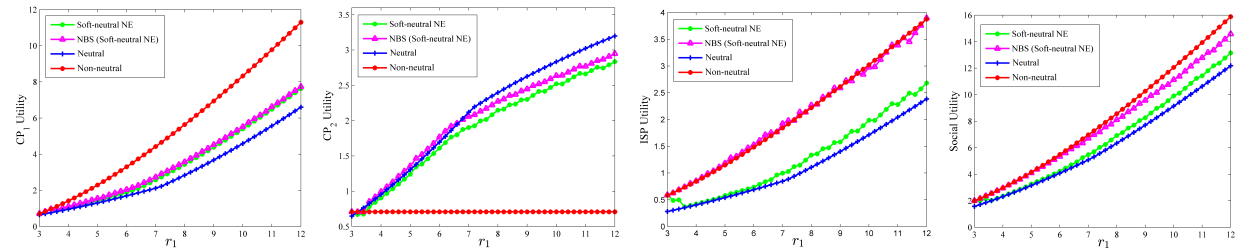


Figure 9: We set $\rho = 0.7$, $c = 1$, $r_2 = 3$ and vary r_1 from 3 to 12.

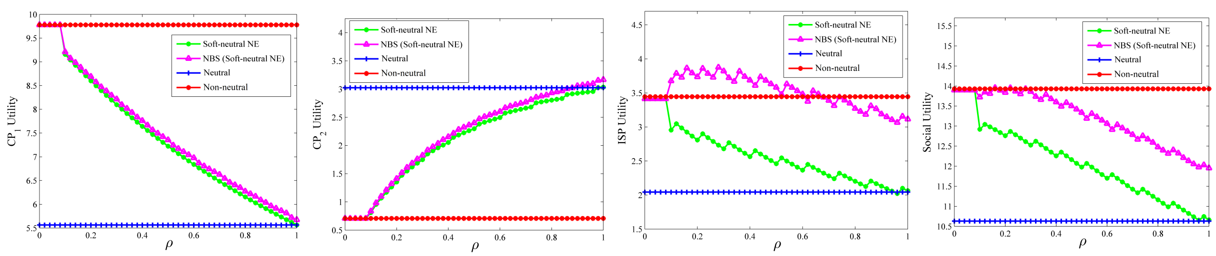


Figure 10: We set $r_1 = 11$, $r_2 = 3$, $c = 1$ and vary ρ from 0 to 1.

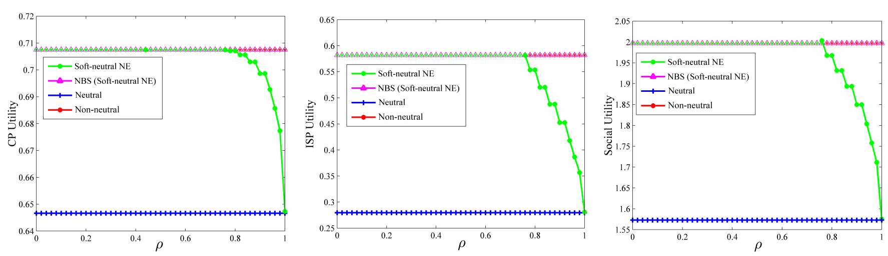


Figure 11: We set $r_1 = r_2 = 3$, $c = 1$ and vary ρ from 0 to 1.

where $A = c\rho(\rho + 1)$, $B = -\rho(\beta_1 r_1 + \beta_2 r_2) + c(\rho + 1)^2$ *and* $C = c(\rho + 1) - (\beta_2 r_2 + \beta_1 r_1 \rho)$

- *If* $a_2^{NN} < \rho a_1^{NN}$, $a_1^{NN} \geq \rho a_2^{NN}$ & $(\beta_1 r_1 + \beta_2 r_2 \rho) - c(\rho + 1) > 0$, *then*
$$a_1^{SN} = \frac{-B + \sqrt{B^2 - 4AC}}{2A}, a_2^{SN} = \rho a_1^{SN}$$
where $A = c\rho(\rho + 1)$, $B = -\rho(\beta_1 r_1 + \beta_2 r_2) + c(\rho + 1)^2$ *and* $C = c(\rho + 1) - (\beta_1 r_1 + \beta_2 r_2 \rho)$.
- *Else,* $((a_1^{SN}, a_2^{SN}) = (0, 0))$.

5.2 Bargaining solution

Having specified the ISP's behavior under the soft-neutrality constraint, we now turn to the next proposed intervention, which is directed at the interaction between the CPs. Specifically, we propose a platform where CPs can interact and bargain with one another to arrive at a vector of revenue sharing contracts to offer to the ISP. We use the classical Nash Bargaining solution (NBS) to capture the outcome of this interaction [21]. Formally, the NBS (β_1^B, β_2^B) is defined as the solution of the following optimization problem:

$$\max_{\beta_1,\beta_2\in[0,1]} \quad (U_{CP_1} - d^{SN}_{CP_1})(U_{CP_2} - d^{SN}_{CP_2}),$$

where $U_{CP_1} = (1 - \beta_i)r_i \log(1 + a^{SN}_i)$ is the utility of CP$_i$ given the ISP behavior specified by Lemma 3. The disagreement point $(d^{SN}_{CP_1}, d^{SN}_{CP_2})$ is taken to correspond to the CP utilities when they act non-cooperatively, i.e., the Nash equilibrium between the CPs.

Numerical Experiments: While an analytical treatment of the soft-neutral regime has eluded us, we illustrate the impact of the proposed interventions via numerical illustrations in Figures 8–11. In these figures, we compare four scenarios: (i) neutral, (ii) non-neutral, (iii) soft-neutral Nash equilibrium (i.e., soft-neutrality without the bargaining framework) and (iv) soft-neutrality NBS (i.e., with both interventions).

Figs. 8 and 9 compares the four scenarios for $\rho = 0.3$ and 0.7, respectively; we plot the utilities of all agents by varying r_1, other parameters being fixed. Note that soft-neutrality produces CP utility that is intermediate between the neutral and non-neutral regimes. Interestingly, ISP utility is highest in the soft-neutral NBS setting. Moreover, social utility is comparable to the non-neutral setting.

Figs. 10 and 11 compare the four scenarios with respect to change in ρ for asymmetric and symmetric CPs respectively. when CPs are asymmetric, can be seen that not only dominant CP utility improves, but ISP and social utility significantly improves in the soft-neutral regime. Utilities are boosted further when CPs bargain alongside the soft-neutrality constraint on the ISP. Once again, for certain range of ρ, ISP's utility is even higher than the non-neutral regime when both interventions are applies. For symmetric CPs, the soft-neutral NBS solution in fact coincides with non-neutral equilibrium; this can also be proved analytically. Most importantly, even when CPs are asymmetric, for a range of ρ values, the social utility of the soft-neutral NBS solution closely matches that under the non-neutral regime.

These (preliminary) results suggest that soft-neutrality provides a promising middle ground between perfect neutrality and no neutrality, balancing the considerations of social utility maximization and a level playing field for content providers on the Internet.

6 CONCLUSIONS AND REGULATORY ISSUES

We studied the problem of revenue sharing between multiple CPs and an ISP on the Internet using the moral hazard framework with multiple principles and a single agent. We compared the revenues of each player and the social utility in a regime where the ISP is forced to put equal effort for all the CPs (neutral) with a regime where there are no such restrictions (non-neutral) on the ISP. Our key takeaway is that everyone is better off, and social utility is higher in the non-neutral regime when the CPs ability to monetize their demand is 'nearly' the same. When there is a significant disparity in the monetization power of the CPs, for the case of two CPs, we showed that non-neutral regime is preferable from the standpoint of the dominant CP (with higher monetizing power), the ISP, and from the standpoint of social utility. On the other hand, the non-dominant CP is benefited by a neutrality stipulation since it gets to 'free-ride' on the contribution made by the dominant CP, the very reason that makes this regime less preferred by the dominant CP.

Our analysis throws up an intriguing dilemma for a regulator—enforcing neutrality brings in parity in the way ISP treats the CPs, but it worsens the social utility and pay-off of all the players compared to the neutral regime if the players act non-cooperatively. It is then interesting to study mechanisms that the regulator can use to induce cooperation among the players so that the social utility and players pay-off is no worse than in the non-neutral regime.

Some possibilities that can be explored are the following ones. If the CPs are working together (like a coalition) and 'bargain' on a transparent platform to come up with mutually binding contracts, the *tragedy of the commons* effect we observe can be avoided. Indeed, in the symmetric setting, it is easy to check that the bargaining solution under the network neutral setting, formalized via the Nash bargaining solution, coincides with the non-neutral setting. On the other hand, when the CPs are asymmetric, the enforcement of *strict neutrality* is itself fundamentally in conflict with social utility maximization. In this case, a weaker notion of neutrality, that stipulates a certain minimum QoS for all services, might be preferable. Thus, we explore a weaker notion of neutrality and call is as *'soft-neutral'*, where a intermediate level of constraint is imposed over ISP. Numerical illustrations show the significant improvement in social utility for the case of asymmetric CPs and when CPs are symmetric soft-neutrality along with Nash bargaining is as good as the non-neutral regime.

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7 APPENDIX

8 PROOF OF THEOREM 1

From CP_i optimization problem, it can be observed that for $r/c < 1$ $U_{\text{CP}_i} = 0$ for all i. Henc e no CP has an incentive to share a fraction of their revenue with the ISP and $\beta_i = 0\ \forall i \in N$ is the equilibirum. Now assume $r/c \geq 1$. For this case the optimial value of β_i will be such that $r\beta_i/c \geq 1$ and the optimization problem of CP_i reduces to

$$\max_{\beta_i \in [0,1]} \ (1-\beta_i) r \log\left(\frac{\beta_i r}{c}\right).$$

The first order optimality condition $\partial U_{CP_i}/\partial \beta_i = 0$ then gives:

$$\log\left(\frac{\beta_i r}{c}\right) = \frac{1-\beta_i}{\beta_i} \ \ \forall i = 1, 2, ..., n.$$

Solving the first order conditions for each CP_i, we get:

$$\frac{1-\beta_i}{\beta_i} = \log\left(\frac{\beta_i r}{c}\right) \implies \frac{1}{\beta_i} = \log\left(\frac{\beta_i re}{c}\right)$$
$$\implies e^{\frac{1}{\beta_i}} = \frac{\beta_i re}{c} \implies \frac{1}{\beta_i} e^{\frac{1}{\beta_i}} = \frac{re}{c}$$

Using the definition of the LamebertW funtion we get

$$\frac{1}{\beta_i} = W\left(\frac{r}{c}e\right) \implies \beta_i = \frac{1}{W\left(\frac{r}{c}e\right)}$$

Hence we get equilibrium contract given in (5).

9 PROOF OF THEOREM 2

Recall the objective of $\text{CP}_i, i \in N$

$$\max_{\beta_i \in [0,1]} \ (1-\beta_i) r \log\left(\max\left(\frac{\sum_{j=1}^n \beta_j r}{nc}, 1\right)\right).$$

First assume that $r/c < 1$. In this case for any given $(\beta_1, \beta_2, \ldots, \beta_{i-1}, \beta_{i+1}, \ldots, \beta_n)$, best response of CP_i is to set $\beta_i = 0$. Thus $\beta_i = 0\ \forall i \in N$ is an equilibrium.
Next consider the case $1 \leq r/c < n$. Fix an $i \in N$ and assume $\beta_j = 0$ for all $j \neq i$. Then the object of CP_i simplifies to

$$\max_{\beta_i \in [0,1]} \ (1-\beta_i) r \log\left(\max\left(\frac{\beta_i r}{nc}, 1\right)\right),$$

and the best response of CP_i is to set $\beta_i = 0$. Hence $\beta_i = 0$ for all $i \in N$ is an equilibrium. We next look for a non-zero equilibrium. By symmetry, it must be such that $\beta_1 = \beta_2 \ldots = \beta_n \in (0, 1]$. Further, at equilibrium it must be the case that $\sum_{i=1}^n \beta_j r/nc \geq 1$, otherwise CPs have incentive to deviate to make their share zero. Writing the first order condition for the optimaization problem of $\text{CP}_i, i \in N$, i.e.,

$$\max_{\beta_i \in [0,1]} \ (1-\beta_i) r \log\left(\frac{\sum_j \beta_j r}{nc}\right),$$

we get

$$\log\left(\frac{\sum_{j=1}^n \beta_j r}{nc}\right) = \frac{1-\beta_i}{\sum_{j=1}^n \beta_j}.$$

Comparing we get $\beta_1 = \beta_2, \ldots, = \beta_n$, let $\beta_i = \beta \forall i$ we have

$$\log\left(\frac{\beta r}{c}\right) = \frac{1-\beta}{n\beta}.$$

Simplyfying the above as earlier in the format of LambertW function we get $\beta = \frac{1}{nW(\frac{r}{nc}e^{1/n})}$.
For the case $r/c \geq n$, $\beta_i = 0, \forall i \in N$ at equilibrium is not arise, however the equilibrium $\beta = \frac{1}{nW(\frac{r}{nc}e^{1/n})}$ still holds.

10 PROOF OF THEOREM 3

Part 1: When $r/c \leq 1, \beta^{NN} = \beta^N = 0$ and the relation $\beta^{NN} \geq \beta^N$ holds trivially. In the range $1 < r/c \leq n$, two equilibria are possible in the neutral regime, $\beta^N = 0$ or $\frac{1}{nW(r/ce^{1/n})}$. If $\beta^N = 0$ is the equilibrium, again the relation holds trivially. Consider the case when $\beta^N = \frac{1}{nW(r/ce^{1/n})}$ is the equilibrium for $1 < r/c$. Define $b := r/c$ and $f(b) = \frac{\beta^{NN}}{\beta^N}$.

$$\lim_{b\to 1} f(b) = \lim_{b \to 1} \frac{nW(\frac{b}{n}e^{\frac{1}{n}})}{W(e)} = \frac{nW(\frac{1}{n}e^{\frac{1}{n}})}{W(e)} = \frac{n.\frac{1}{n}}{1} = 1 \ \text{(using } x = W(xe^x))$$

The limit holds as the equlibrium definition holds for all $b > 1$ and W is continuous at $b = 1$. Also, $f(b)$ is monotonically increasing in b forall $b > 1$ as

$$\frac{\partial f(b)}{\partial b} = \frac{nW(be^{\frac{1}{n}}/n)}{bW(be)}\left[\frac{W(be) - W(be^{\frac{1}{n}}/n)}{(1+W(be))(1+W(be^{\frac{1}{n}}/n))}\right] > 0 \forall b > 1$$

Hence $\beta^{NN} > \beta^N$. It holds similarly for the case $r/c > n$.
Part 2: Since investment decision by ISP is monotonically increasing in the share in gets fromt the CPs (from Eqns. (6) and (11), by Part 1 it is clear that ISP make more investment in non-neutral regime as compared to neutral regime.
Part 3: In both non-neutral and neutral regime equilibrium effort, a for given β is $a + 1 = \max\left(\frac{\beta r}{c}, 0\right)$.

Now, in both non-neutral and neutral regimes, each CP's utility at equilibrium is the same function given by $(1-\beta)r \log\left(\max\left(\frac{\beta r}{c}, 0\right)\right)$ which is concave in $\beta \in (c/r, 1)$ and from Part 1 we have that $\beta^N \leq \beta^{NN}$. This implies that $U_{CP}^N \leq U_{CP}^{NN}$ (seen Fig. 12)
Part 4: $U_{ISP}^{NN} = [n\beta^{NN} r \ \log(a^{NN}+1) - nc(a^{NN})]$
Substituting the value of $a^{NN}(\beta^{NN}) = \frac{\beta^{NN} r}{c} - 1$ in the second term of above expression, we get:

$$U_{ISP}^{NN} = \{n\beta^{NN} r \left[\ \log(a^{NN}+1) - 1\right] + nc\} \tag{16}$$

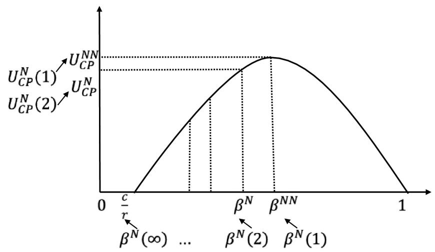

Figure 12: Utility of CP vs β

Similarly,

$$U_{ISP}^{N} = [n\beta^N r\left[\log(a^N + 1) - 1\right] + nc] \tag{17}$$

From Part 1 & 2, we have $\beta^{NN} \geq \beta^N$ & $a^{NN} \geq a^N$, respectively. Comparing (16) & (17) gives $U_{ISP}^{NN} \geq U_{ISP}^{N}$.

11 PROOF OF THEOREM 4

Part 1: Considering n to be continuous variable,

$$\frac{\partial \beta^N(n)}{\partial n} = \frac{1}{n^2 W\left(\frac{r}{nc}e^{1/n}\right)}\left[-1 + \frac{1+\frac{1}{n}}{1 + W\left(\frac{r}{nc}e^{1/n}\right)}\right]$$

Now, $\beta^N(n)$ decreases with n *iff* $\frac{\partial \beta^N}{\partial n} < 0$

$$\iff \left[-1 + \frac{1+\frac{1}{n}}{1 + W\left(\frac{r}{nc}e^{1/n}\right)}\right] < 0 \iff \left[\frac{1+\frac{1}{n}}{1 + W\left(\frac{r}{nc}e^{1/n}\right)}\right] < 1$$

$$\iff 1 + \frac{1}{n} < 1 + W\left(\frac{r}{nc}e^{1/n}\right) \iff 1 < nW\left(\frac{r}{nc}e^{1/n}\right)$$

it holds as $\beta^N(n) = \frac{1}{nW\left(\frac{r}{nc}e^{1/n}\right)} < 1 \implies nW\left(\frac{r}{nc}e^{1/n}\right) > 1$.

Part 2: Effort of ISP for each CP is decreasing following directly as β^N is decreasing in n. The total effort of ISP is $A^N(n) = na^N(n) = n\left(\frac{\beta^N r}{c} - 1\right)$. In the following we show that $A^N(n+1) > A^N(n)$ for any n. We have

$$A^N(n+1) > A^N(n) \iff (n+1)\beta^N(n+1) - n\beta^N(n) > \frac{c}{r} \tag{18}$$

We prove that the above inequality holds in two part.

Part (i): We first prove that $g(n) = n\beta^N(n)$ is concave in n, which implies the difference $(n+1)\beta^N(n+1) - n\beta^N(n)$ shrinks as n increases. Now,

$$g(n) = n\beta^N(n) = \frac{1}{W\left(\frac{b}{n}e^{\frac{1}{n}}\right)}; \text{ where } b = \frac{r}{c}$$

It is clear that $g(n)$ is increasing in n. Treating n as continuous variable, we have

$$\frac{\partial f(n)}{\partial n} = \frac{n+1}{n^2 W\left(\frac{b}{n}e^{\frac{1}{n}}\right)\left(1 + W\left(\frac{b}{n}e^{\frac{1}{n}}\right)\right)} > 0$$

$$\frac{\partial^2 f(n)}{\partial n^2} = \frac{1}{n^4 W\left(\frac{b}{n}e^{\frac{1}{n}}\right)\left(1 + W\left(\frac{b}{n}e^{\frac{1}{n}}\right)\right)^2} \times$$

$$\left\{(n+1)^2 \frac{\left(1 + 2W\left(\frac{b}{n}e^{\frac{1}{n}}\right)\right)}{\left(1 + W\left(\frac{b}{n}e^{\frac{1}{n}}\right)\right)} - n(n+2)\left(1 + W\left(\frac{b}{n}e^{\frac{1}{n}}\right)\right)\right\}$$

Now, $f(n)$ is strictly concave in n *iff* $\frac{\partial^2 f(n)}{\partial n^2} < 0$

$$\iff \frac{(n+1)^2}{n(n+2)} < \frac{\left(1 + W\left(\frac{b}{n}e^{\frac{1}{n}}\right)\right)^2}{\left(1 + 2W\left(\frac{b}{n}e^{\frac{1}{n}}\right)\right)}$$

After cross multiplying and expanding, we get

$$\frac{1 + 2W\left(\frac{b}{n}e^{\frac{1}{n}}\right)}{nW\left(\frac{b}{n}e^{\frac{1}{n}}\right) + 2W\left(\frac{b}{n}e^{\frac{1}{n}}\right)} < nW\left(\frac{b}{n}e^{\frac{1}{n}}\right)$$

We know that $1/\beta^N = nW\left(\frac{a}{n}e^{\frac{1}{n}}\right) > 1$ at equilibrium,therefore LHS<1 and RHS>1. Thus, the above inequality holds .

Part (ii): Now, we show that $(n+1)\beta^N(n+1) - n\beta^N(n) \to c/r$ as $n \to \infty$. Consider asymptotic expansion of LambertW function, $W(x) = x - x^2 + o(x^2) = x(1 - x + o(x))$

$$\frac{1}{W(x)} = \frac{1}{x(1 - x + o(x))} = \frac{1}{x}\cdot(1 + x + o(x)) = \frac{1}{x} + 1 + o(1)$$

second equality comes by using $\frac{1}{1-x} = 1 + x + o(x)$.Now,

$$f(n) = n\beta^N(n) = \frac{1}{W\left(\frac{r}{nc}e^{\frac{1}{n}}\right)} = \frac{nc}{re^{\frac{1}{n}}} + 1 + o(1)$$

$$= \frac{nc}{r}\left(1 - \frac{1}{n} + o\left(\frac{1}{n}\right)\right) + 1 + o(1) = \frac{nc}{r} - \frac{c}{r} + 1 + o(1) \tag{19}$$

third equality comes by using $e^{-x} = 1 - x + o(x)$.Therefore,

$$f(n+1) - f(n) = \frac{(n+1)c}{r} - \frac{nc}{r} \to \frac{c}{r} \quad \text{as} \quad n \to \infty$$

Part 3: For any n each CP's utility for the given a at equilibrium, is given by $(1-\beta)r\log\left(\max\left(\frac{\beta r}{c}, 0\right)\right)$.

We know that the above function is concave in $\beta \in (c/r, 1]$ and from Part 1, as n increases $\beta^N(n)$ decreases and approaches c/r when $n \to \infty\left(\frac{1}{nW\left(\frac{r}{nc}e^{\frac{1}{n}}\right)} = \frac{1}{n\left(\frac{r}{nce^{1/n}} - \left(\frac{r}{nce^{1/n}}\right)^2 - o\left(\frac{1}{n^2}\right)\right)} \to \frac{c}{r}, \text{as} n \to \infty\right)$.

This implies that $U_{CP}^{N}(n)$ decreases as n increases (see Fig. 12). Also it can be seen that as $n \to \infty, U_{CP}^{N} \to 0$.

Part 4:

$$\begin{aligned} U_{ISP}^{N} &= n(\beta^N r\log(a^N + 1) - ca^N) \\ &= n\left(\beta^N r\log\left(\frac{\beta^N r}{c}\right) - c\left(\frac{\beta^N r}{c} - 1\right)\right) \\ &= r + nc - (n+1)\beta^N r \end{aligned}$$

Utility of ISP from each CP is

$$\frac{U_{ISP}^{N}}{n} = \beta^N r\log(a^N + 1) - ca^N \tag{20}$$

Substituting the value of $a^N = \frac{\beta^N r}{c} - 1$ in the second term, we get $\frac{U_{ISP}^{N}}{n} = \beta^N r\left(\log(a^N + 1) - 1\right) + c$. Since β^N ans a^N decreases with increase in n, it is clear from above expression that utility of ISP from each CP also decreases with increase in number of CPs.

Now, $U_{ISP}^N(n)$ increases with increase in n *iff*

$$\frac{U_{ISP}^N(n)}{\partial n} > 0 \quad \text{(considering } n \text{ to be continuous)}$$

$$c - \frac{1}{n^2 W\left(\frac{r}{nc}e^{\frac{1}{n}}\right)}\left(\frac{n^2+n+1-nW\left(\frac{r}{nc}e^{\frac{1}{n}}\right)}{n\left(1+W\left(\frac{r}{nc}e^{\frac{1}{n}}\right)\right)}\right) r > 0$$

$$\frac{1}{n^2 W\left(\frac{r}{nc}e^{\frac{1}{n}}\right)}\left(\frac{n^2+n+1-nW\left(\frac{r}{nc}e^{\frac{1}{n}}\right)}{n\left(1+W\left(\frac{r}{nc}e^{\frac{1}{n}}\right)\right)}\right) < \frac{c}{r}$$

Since, $c/r > 0$, we have

$$\frac{1}{n^2 W\left(\frac{r}{nc}e^{\frac{1}{n}}\right)}\left(\frac{n^2+n+1-nW\left(\frac{r}{nc}e^{\frac{1}{n}}\right)}{n\left(1+W\left(\frac{r}{nc}e^{\frac{1}{n}}\right)\right)}\right) < 0$$

$$\iff n^2+n+1-nW\left(\frac{r}{nc}e^{\frac{1}{n}}\right) < 0.$$

12 ASYMMETRIC CASE: EQUILIBRIUM CONTRACTS FOR $n > 2$

THEOREM 6. *In the non-neutral regime equilibrium contract for CP_i is given by*

$$\beta_i^{NN} = \begin{cases} 0 & \text{if } \frac{r_i}{c} < 1, \\ \frac{1}{W(\frac{r_i}{c}e)} & \text{if } \frac{r_i}{c} \geq 1, \end{cases} \tag{21}$$

Further, the effort levels of the ISP for are given by

$$a_i^{NN} = \max\left(\frac{\beta_i^{NN} r_i}{c} - 1, 0\right) \quad \forall i = 1, 2, \cdots n. \tag{22}$$

THEOREM 7. *In the neutral regime, each $CP_i, i = 1, 2, \cdots, n$ shares a positive fraction of the revenue at equilibrium with the ISP only if $r_i/c > 1 \forall i = 1, 2, \cdots, n$ and $r_1, r_2, ..., r_n$ are close enough to each other. Specifically, the equilibrium contract is as follows*

$$\beta_i^N = \frac{\sum_{j=1}^n r_j}{n^2 r_i W\left(\frac{\sum_{j=1}^n r_j}{n^2 c}e^{\frac{1}{n}}\right)} - \frac{\sum_{j=1, j\neq i}^n r_j - (n-1)r_i}{nr_i}; \forall i = 1, 2, ..., n. \tag{23}$$

and the equilibrium effort is $a^N = \left(\frac{\sum_{j=1}^n \beta_j r_j}{nc} - 1\right)$.

When $r_1 >> r_2$, only CP_1 shares positive fraction at equilibrium, and the equilibrium contract is given as follows:

$$\beta_1^N = \frac{1}{W\left(\frac{r_1}{nc}e\right)}, \& \beta_i^* = 0, \forall i = 2, 3, ..., n \tag{24}$$

and the equilibrium effort is $a^N = \left(\frac{\beta_1 r_1}{nc} - 1\right)$

Proof: Substituting the best action of ISP determined in CP$_i$'s optimization problem, we get:

$$\max_{\beta_i} \quad (1-\beta_i) r_i \log\left(\max\left(\frac{\sum_{j=1}^n \beta_j r_j}{nc}, 1\right)\right)$$

First order necessary condition for CP$_i$ gives

$$\frac{(1-\beta_i)r_i}{\sum_{j=1}^n \beta_j r_j} - \log\left(\frac{\sum_{j=1}^n \beta_j r_j}{nc}\right) = 0 \;\forall\; i = 1, 2, ..., n \tag{25}$$

Comparing these set of eqns, we get,

$$(1-\beta_1)r_1 = (1-\beta_2)r_2 = ... = (1-\beta_n)r_n$$

$$\implies \beta_i r_i = \beta_j r_j + r_i - r_j; \;\forall\; i = 1, 2, ..., n; i \neq j$$

$$\therefore \sum_{j=1}^n \beta_j r_j = n\beta_i r_i + \sum_{j=1, j\neq i}^n r_j - (n-1)r_i \forall\; i = 1, 2, ..., n$$

Substituting, we get

$$\frac{(1-\beta_i)r_i}{n\beta_i r_i + \sum_{j=1,j\neq i}^n r_j - (n-1)r_i} = \log\left(\frac{n\beta_i r_i + \sum_{j=1,j\neq i}^n r_j - (n-1)r_i}{nc}\right) \tag{26}$$

Adding $1/n$ to both the sides of eqn.(26), we get

$$\frac{\sum_{j=1}^n r_j}{n\left(n\beta_i r_i + \sum_{j=1,j\neq i}^n r_j - (n-1)r_i\right)} = \log\left(\frac{n\beta_i r_i + \sum_{j=1,j\neq i}^n r_j - (n-1)r_i}{nc}\right) + \log e^{\frac{1}{n}}$$

Rearranging and solving, we get

$$\beta_i = \frac{\sum_{j=1}^n r_j}{n^2 r_i W\left(\frac{\sum_{j=1}^n r_j}{n^2 c}e^{\frac{1}{n}}\right)} - \frac{\sum_{j=1,j\neq i}^n r_j - (n-1)r_i}{nr_i}; i = 1, 2, ..., n$$

Since, $r_1 > r_2$, $\beta_1 > 0$, however, $\beta_2, \beta_3, ..., \beta_n$ in above expression can tale negative value. therefore, the above solution holds only if r_i's are sufficiently close s.t. above solution is positive $\forall i = 1, 2, ..., n$. Else, $\beta_2, \beta_3, ..., \beta_n = 0$, and β_1 is obtained from $\frac{(1-\beta_1)}{\beta_1} - \log\left(\frac{\beta_1 r_1}{nc}\right) = 0$. Solution of which is $\beta_1 = \frac{1}{W(\frac{r_1}{nc}e)}$.

13 PROOF OF PROPOSITION 1

Part 1: For all $r_1 < r_1$, differentiating β_2^N w.r.t r_1, we get

$$\frac{\partial \beta_2^N}{\partial r_2} = \frac{1}{4\left(1+W\left(r_2\frac{r_1+r_2}{4c}e^{0.5}\right)\right)} - \frac{1}{2} \leq 0 \;\forall\, r_1 \geq r_2$$

which implies decreasesβ_2^N with increase in r_1.
And for $r_1 > r_1^*$, $\beta_2^N = 0$. And $\beta_2^{NN} = \frac{1}{W(\frac{r_2}{c}e^{0.5})} > 0$ which remain unchanged with increase in r_1. Also at $r_1 = r_2$ (symmetric case), $\beta_2^{NN} \geq \beta_2^N$. Thus, $\beta_2^{NN} \geq \beta_2^N \;\forall\, r_1 \geq r_2$.
Part 2: It is clear from the expression of β_1^{NN} that it is decreasing in r_1 Now, there exist some $r_1 > r_1^*$ for which β_1^N is $\beta_1^N = \frac{1}{W(\frac{r_1}{2c}e)}$,which also decreases with increase in r_1. Also, for $r_1 > r_1^*$, $\beta_1^N > \beta_1^{NN}$. Now, consider the case when $r_1 \leq r_1^*$ where

$$\beta_1^N = \frac{r_1+r_2}{4r_1 W\left(\frac{r_1+r_2}{4c}e^{0.5}\right)} - \frac{r_2-r_1}{2r_1}$$

Now, differentiating β_1^N w.r.t r_1, we get:

$$\frac{\partial \beta_1^N}{\partial r_1} = \frac{\left(1 + W\left(\frac{r_1+r_2}{4c}e^{0.5}\right)\right)\left(2W\left(\frac{r_1+r_2}{4c}e^{0.5}\right) - 1\right) - r_1}{4r_1^2 W\left(1 + W\left(\frac{r_1+r_2}{4c}e^{0.5}\right)\right)}$$

And it can take both negative and positive values depending upon value of r_2. Therefore, it if not apparent that whether β_1^N increases or decreases when $r_1 < r_1^*$.

14 PROOF OF PROPOSITION 2

Part 1: Utility of CP_1 in non-neutral regime is given by

$$U_{CP_1}^{NN} = (1 - \beta_1^{NN})r_1 \log\left(\frac{\beta_1^{NN} r_1}{c}\right) = \frac{(1 - \beta_1^{NN})^2}{\beta_1^{NN}} r_1$$

(Using first order condition $\frac{(1-\beta_1^{NN})}{\beta_1^{NN}} = \log\left(\frac{\beta_1^{NN} r_1}{c}\right)$)

Similarly, utility of CP_1 $\forall r_1 > r_1^*$ in neutral regime is

$$U_{CP_1}^{N} = (1 - \beta_1^{N})r_1 \log\left(\frac{\beta_1^{N} r_1}{2c}\right) = \frac{(1 - \beta_1^{N})^2}{\beta_1^{N}} r_1$$

We know $\forall r_1 > r_1^*, \beta_1^N > \beta_1^{NN}$.

$$\implies (1 - \beta_1^N)^2 < (1 - \beta_1^{NN})^2 \text{ and } \frac{1}{\beta_1^N} < \frac{1}{\beta_1^{NN}}$$

$$\implies \frac{(1 - \beta_1^N)^2}{\beta_1^N} r_1 < \frac{(1 - \beta_1^{NN})^2}{\beta_1^{NN}} r_1, \text{thus, } U_{CP_1}^N < U_{CP_1}^{NN}$$

Part 2: Utility of CP_2 in non-neutral regime is given by

$$U_{CP_2}^{NN} = (1 - \beta_2^{NN})r_2 \log\left(\frac{\beta_2^{NN} r_2}{c}\right) = \frac{(1 - \beta_2^{NN})^2}{\beta_2^{NN}} r_2$$

(Using first order condition $\frac{(1-\beta_2^{N})}{\beta_2^{N}} = \log\left(\frac{\beta_2^{N} r_1}{c}\right)$)

Similarly, utility of CP_2 $\forall r_1 \geq r_1^*$ in non-neutral regime is

$$U_{CP_2}^{N} = (1 - \beta_2^{N})r_2 \log\left(\frac{\beta_1^{N} r_1 + \beta_2^{N} r_2}{2c}\right) = \frac{(1 - \beta_1^{N})}{\beta_1^{N}} r_2$$

$$\text{Now, } U_{CP_2}^{NN} \leq U_{CP_2}^{N} \text{ iff } \frac{(1 - \beta_2^{NN})^2}{\beta_2^{NN}} \leq \frac{(1 - \beta_1^{N})}{\beta_1^{N}}$$

We know that when $r_1 > r_1^*$, β_1^N decreases with increase in r_1. Thus, RHS of above inequality is increasing in r_1. However, β_2^{NN} remain unchanged with increase in r_1, implying that LHS of the above inequality is constant. Therefore, there exist some r_1, beyond which the above inequality holds. We know that $U_{CP_2}^{NN}$ remain constant with increase in r_1.

15 PROOF OF LEMMA 1

Utility of ISP in non-neutral regime is given by

$$U_{ISP}^{NN} = (1 - 2\beta_1^{NN})r_1 + (1 - 2\beta_2^{NN})r_2 + 2c$$

(Using first order condition $\frac{(1-\beta_i^{N})}{\beta_i^{N}} = \log\left(\frac{\beta_i^{N} r_i}{c}\right); i = 1, 2$)

Similarly, in neutral regime for $r_1 \geq r_1^*$, it is given by

$$U_{ISP}^{N} = (1 - 2\beta_1^{N})r_1 + 2c$$

$$U_{ISP}^{NN} \geq U_{ISP}^{N} \textit{ iff } 2(\beta_1^N - \beta_1^{NN})r_1 \geq -(1 - 2\beta_2^{NN})r_2$$

$$\iff 2r_1\left(\frac{1}{W(\frac{r_1}{2c}e)} - \frac{1}{W(\frac{r_1}{c}e)}\right) \geq \left(2\frac{1}{W(\frac{r_2}{c}e)} - 1\right) r_2$$

LHS in increasing in r_1, however RHS remain unchanged. Therefore, there must exist some $r_1 > r_1^*$ say r_1^bs.t. for all $r_1 > r_1^b$ the above inequality holds. Also, plot shows that ISP is always better off in non-neutral regime.

16 PROOF OF LEMMA 2

For $r_1, r_2 > c$, we have

$A^N \geq A^{NN}$ *iff* $r_1\left(\frac{1}{W(\frac{r_1}{2c}e)} - \frac{1}{W(\frac{r_1}{c}e)}\right) \geq \frac{r_2}{W(\frac{r_2}{c}e)}$

LHS of above inequality is increasing in r_1 and RHS remains unchanged. With increase in r_1 for fixed r_2, the above inequality will start holding for some large value of r_1. Thus, the above inequality will hold for some large enough $r_1^a >> r_1^*$.

For $r_1 > c,\ r_2 \leq c$, we have

$A^N \geq A^{NN}$ *iff* $\frac{r_1}{c}\left(\frac{1}{W(\frac{r_1}{2c}e)} - \frac{1}{W(\frac{r_1}{c}e)}\right) \geq 1$

LHS of above inequality is increasing in r_1 and RHS remains unchanged. With increase in r_1 for fixed c, the above inequality will start holding for some large value of r_1. Thus, the above inequality will hold for some large enough $r_1^a >> r_1^*$.