Network Speed Scaling

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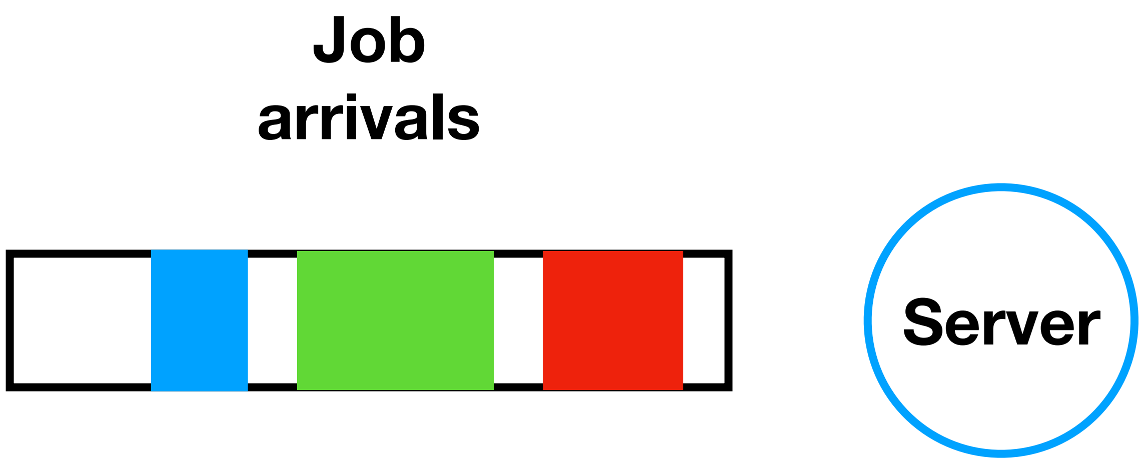
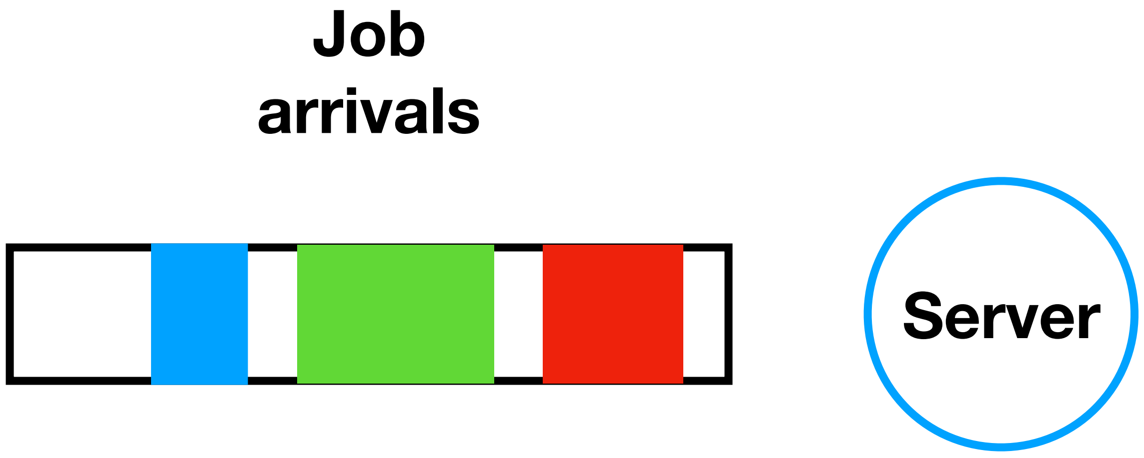
Joint work Jayakrishnan Nair- IIT-Bombay

Speed Scaling 101

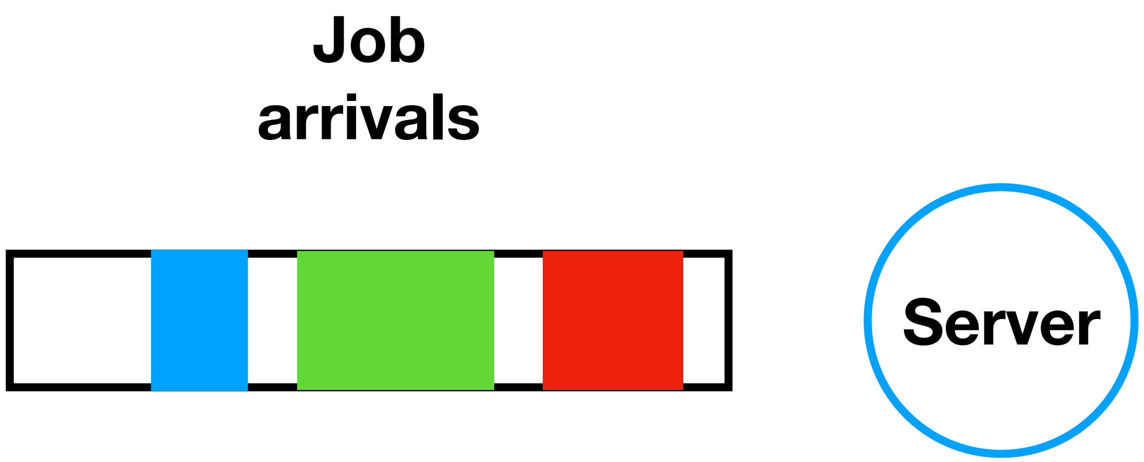


Speed Scaling 101



Speed **tuneable** server

Speed Scaling 101



Speed **tuneable** server

Speed s Power $P(s)$

e.g. $P(s) = s^{\alpha}$

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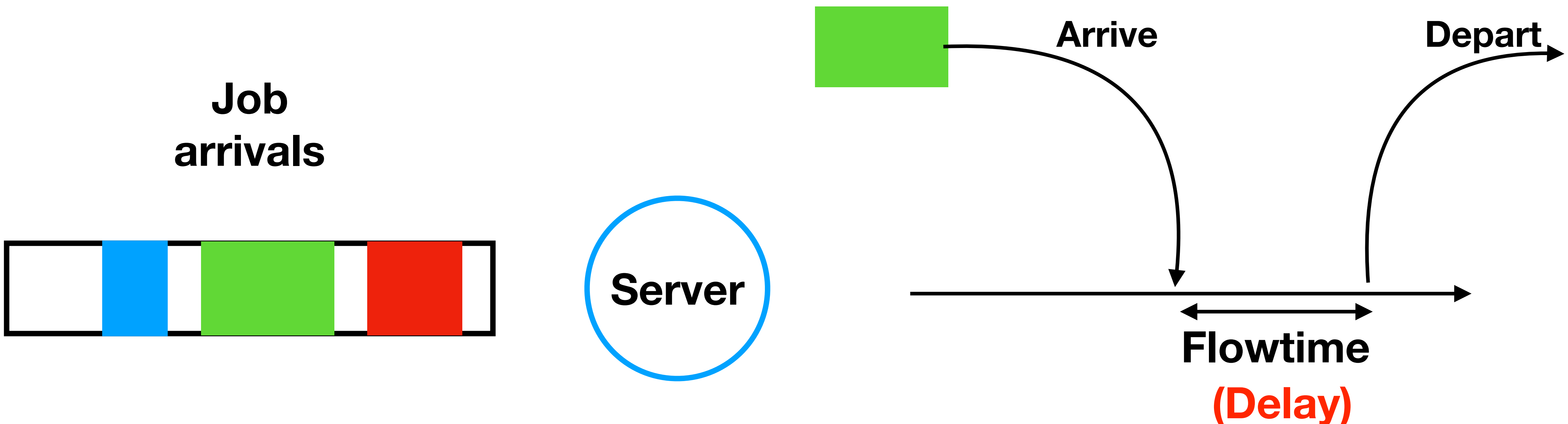

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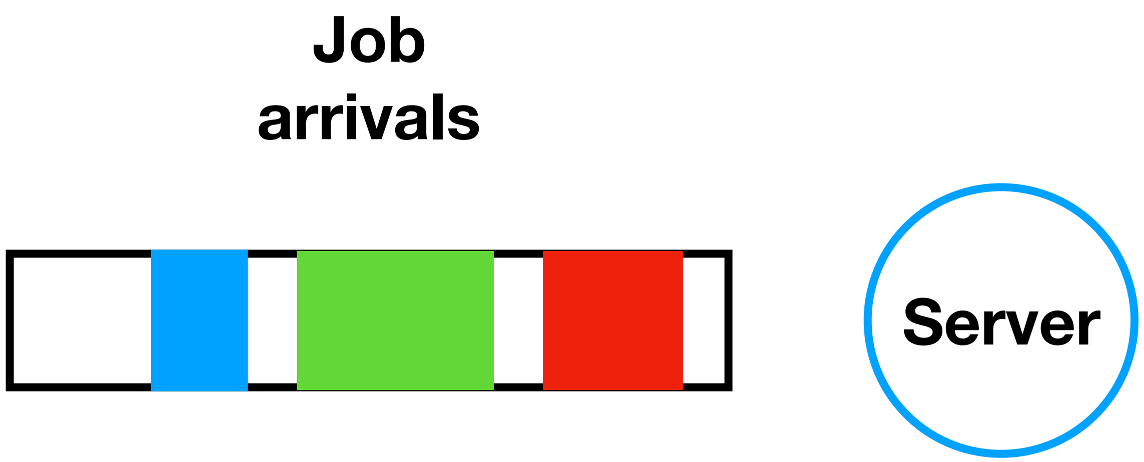

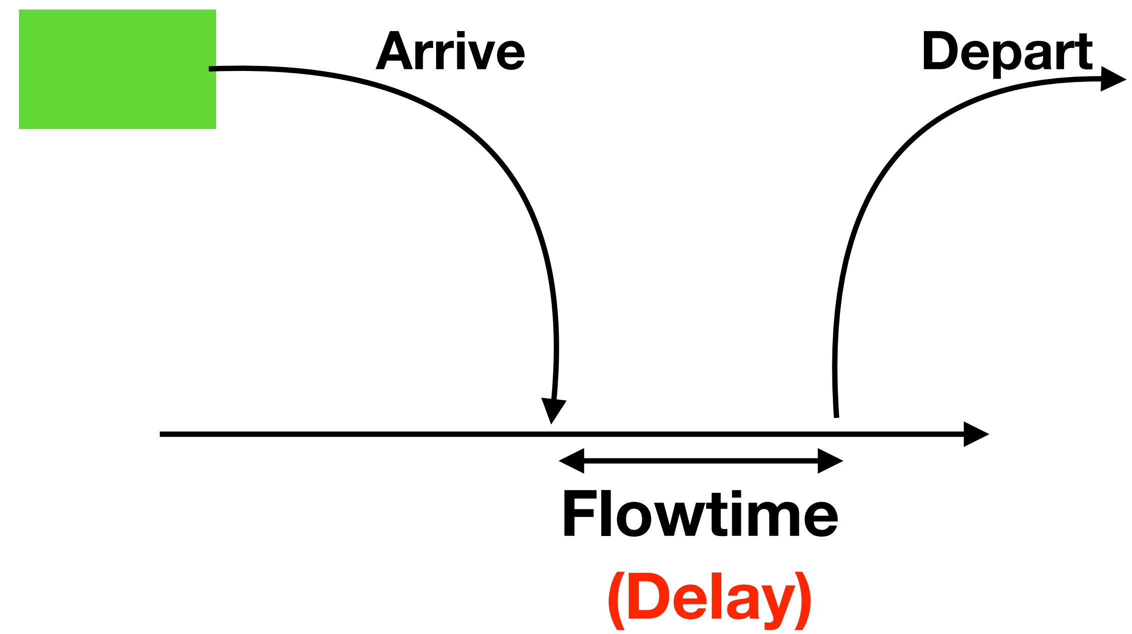

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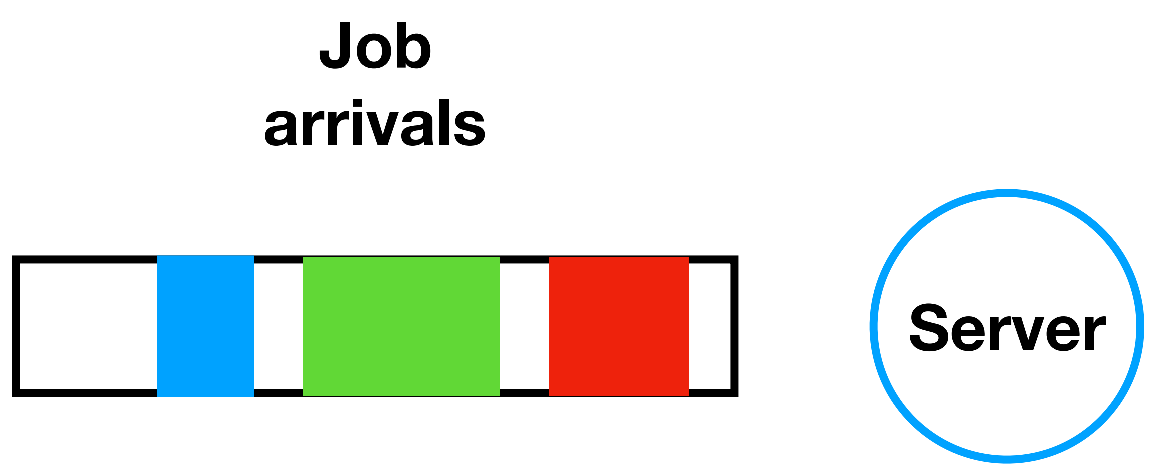

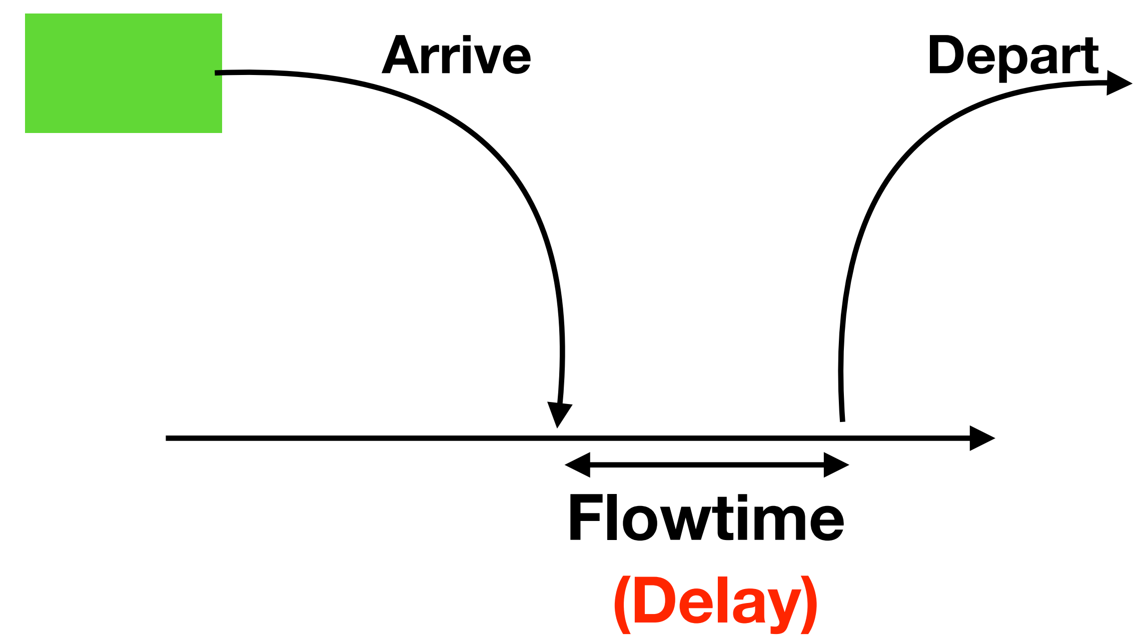

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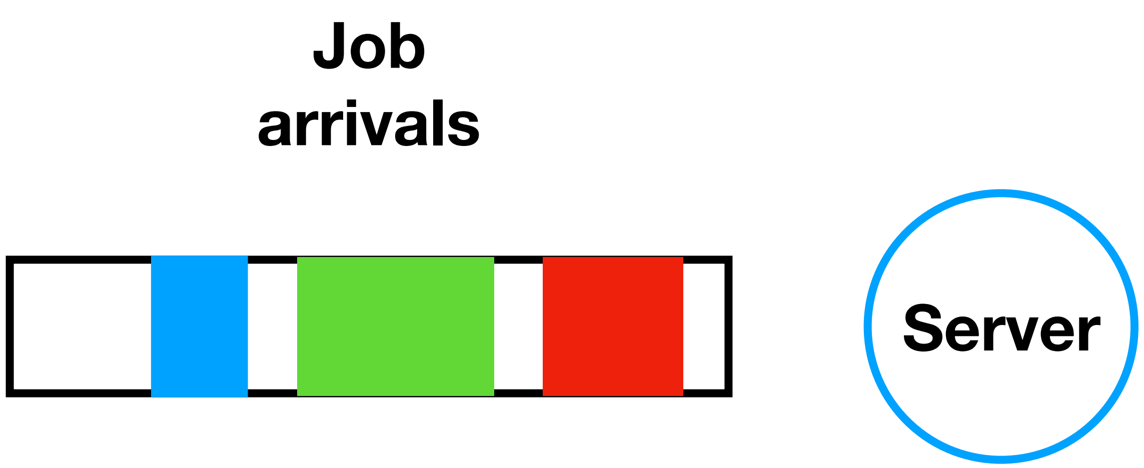
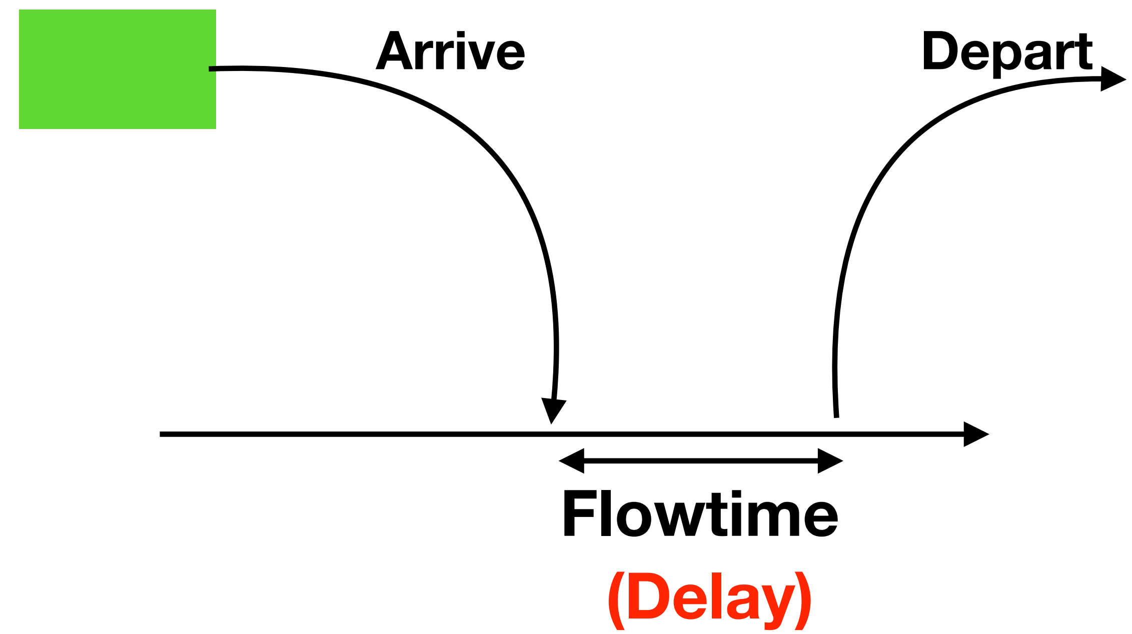
$$\int n(t)dt \quad + \int P(s(t))dt$$

n(t) **number of outstanding jobs at time t**

Speed Scaling 101





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Decisions

Speed, Scheduling

Speed Scaling in Networks

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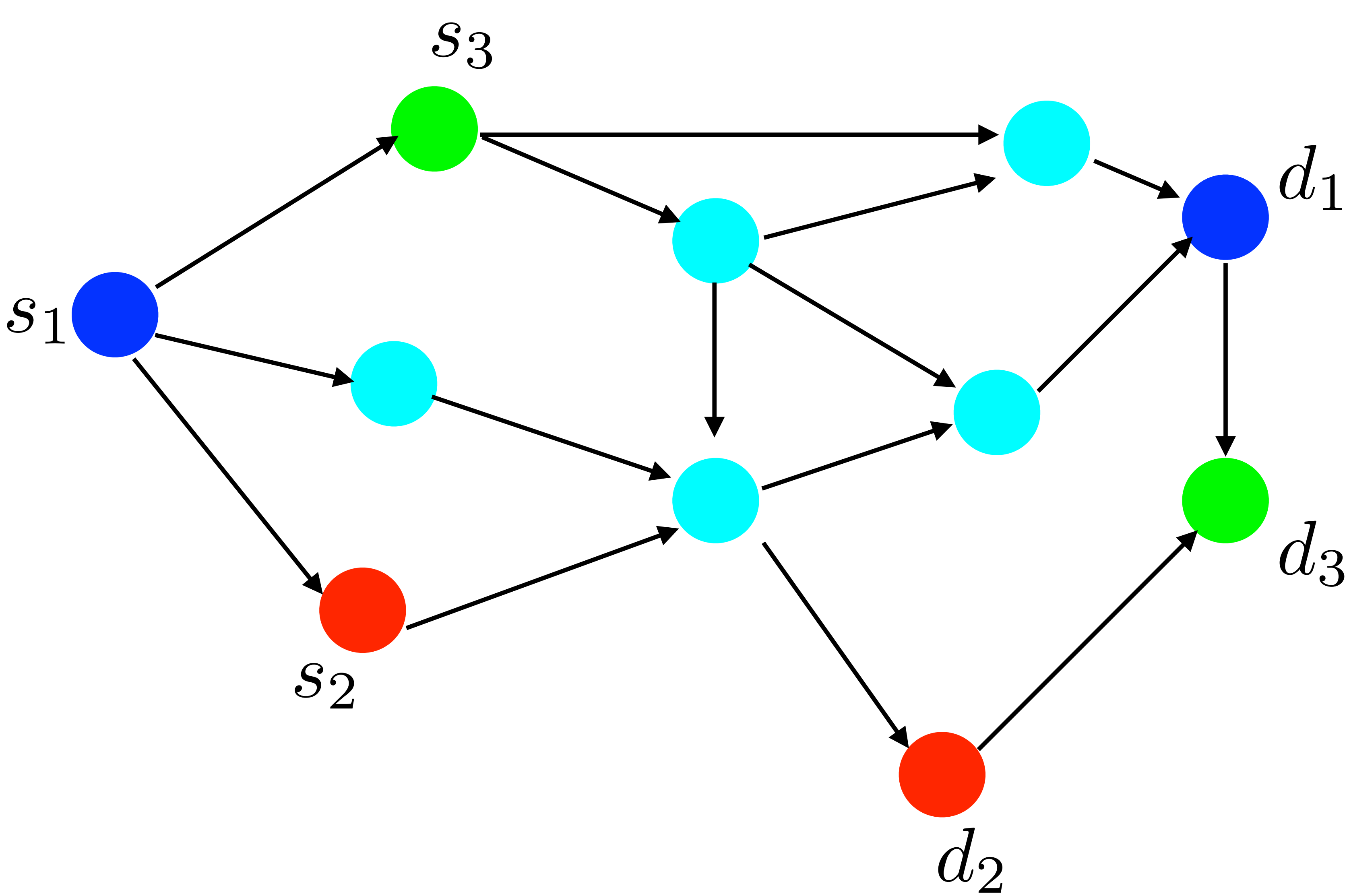

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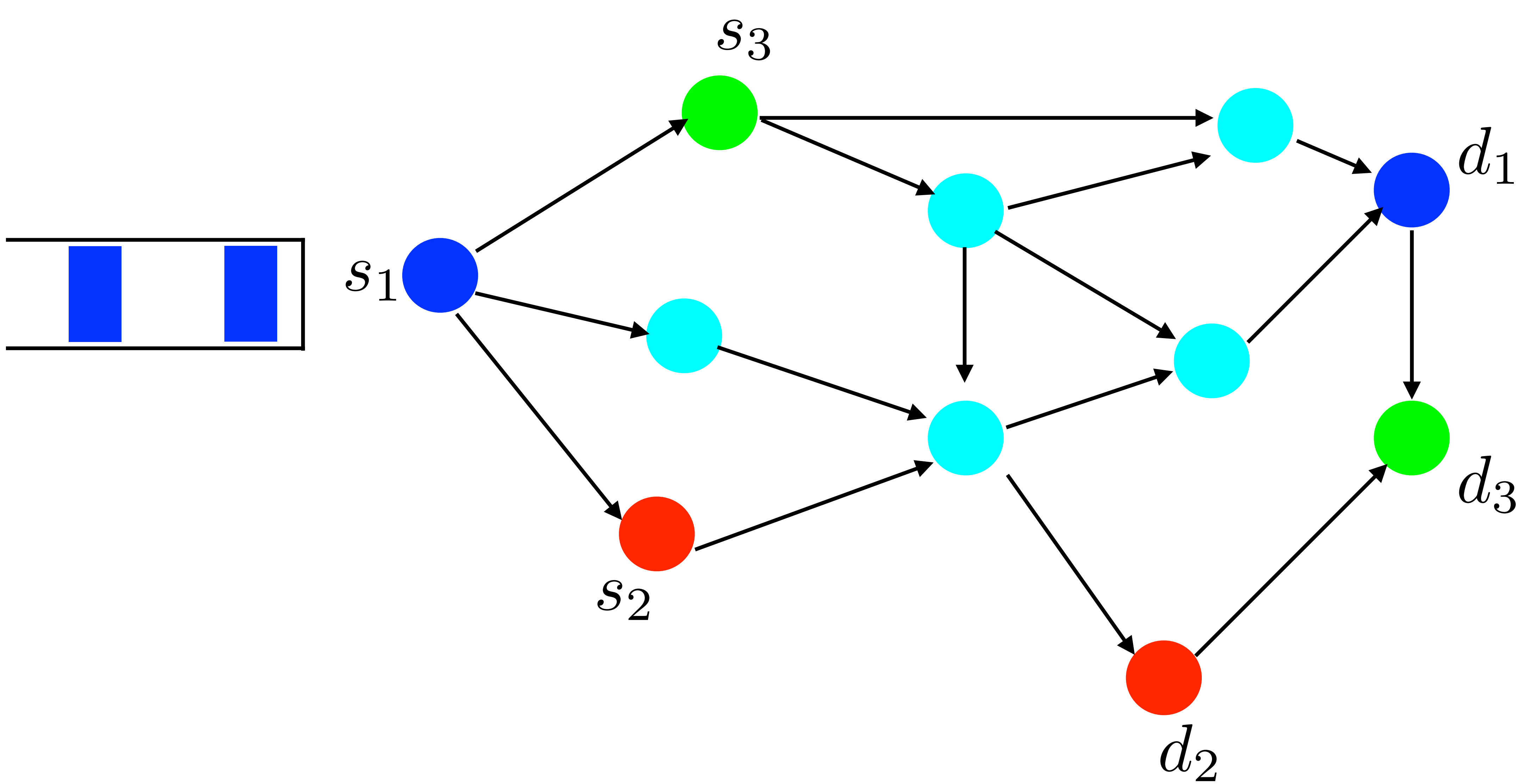

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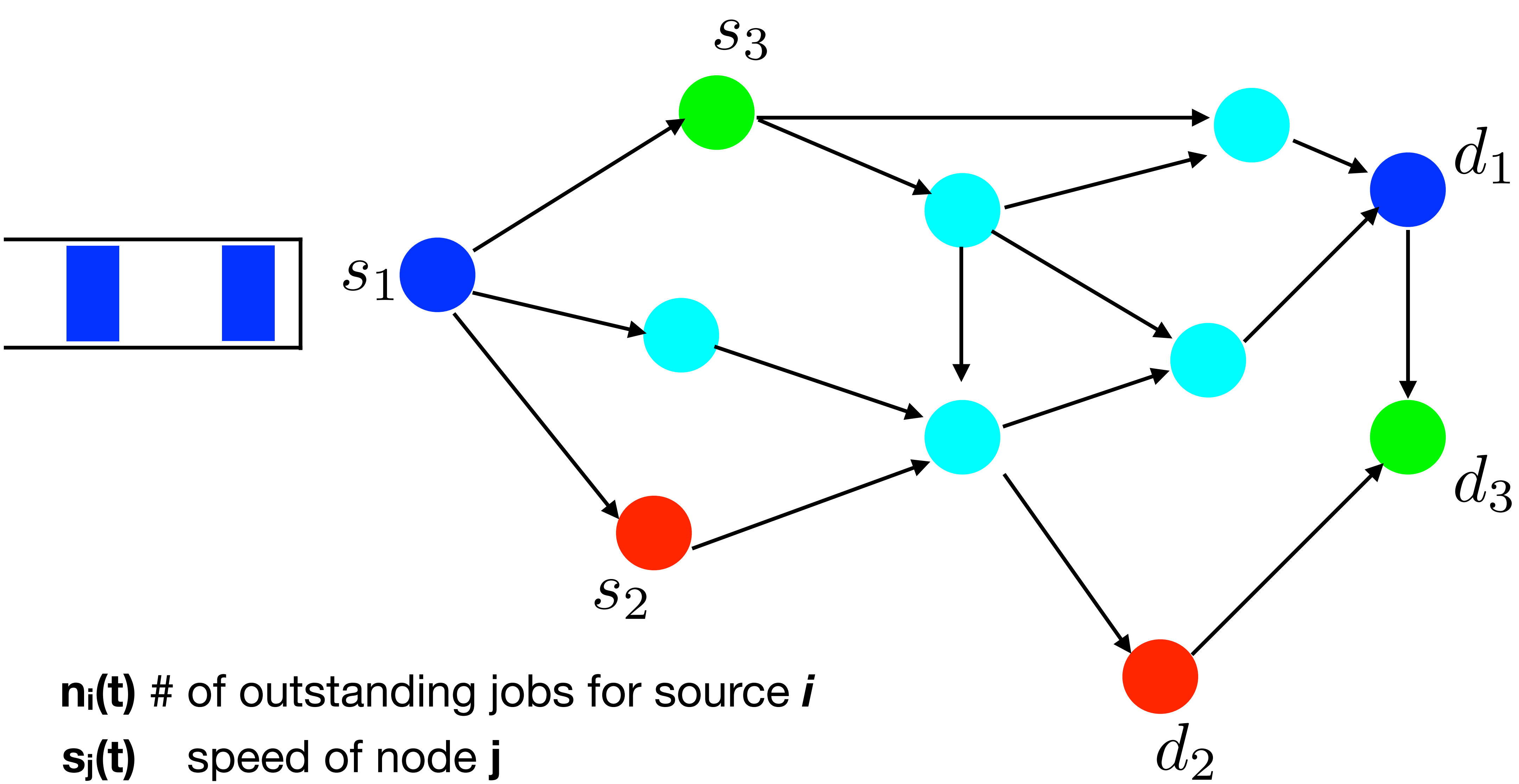

n_i(t) # of outstanding jobs for source ***i***

s_j(t) speed of node **j**

Obj:

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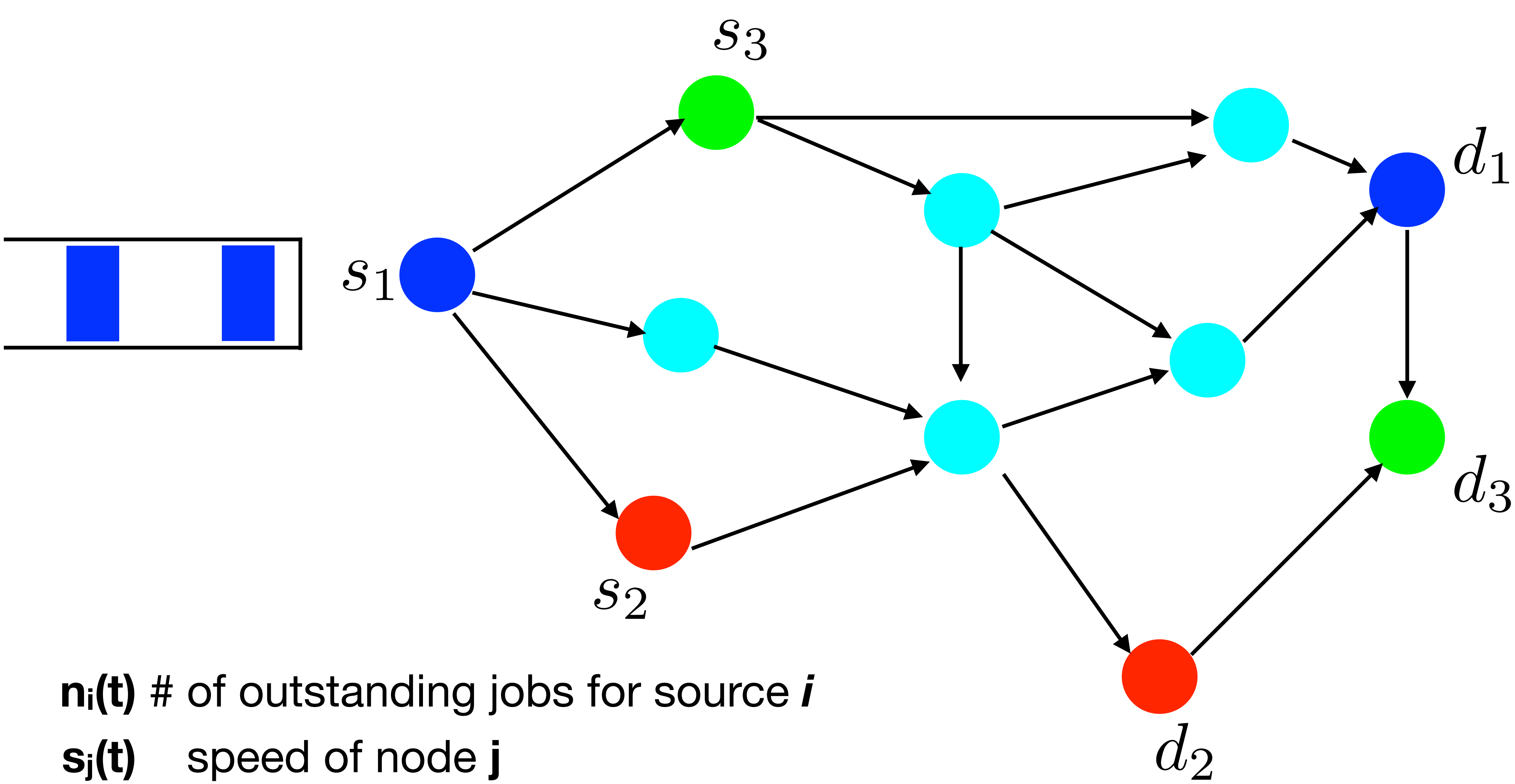
Flow time Energy

Speed Scaling in Networks



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Flow time Energy

Decisions **Speed, Routing, Scheduling**

Online Algorithms

Online Algorithms

Algorithms with causal information

Online Algorithms

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Centralized

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Competitive ratio

Online Algorithms

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Competitive ratio ratio of the cost of an **online** and the **offline Opt** algorithm

Online Algorithms

Algorithms with causal information

Centralized

Competitive ratio ratio of the cost of an **online** and the **offline Opt** algorithm

Worst Case Input

$$r_{\mathsf{ON}} = \max_{\sigma} \frac{v_{\mathsf{ON}}(\sigma)}{v_{\mathsf{OPT}}(\sigma)}$$

Input

Online Algorithms

Algorithms with causal information

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Competitive ratio ratio of the cost of an **online** and the **offline Opt** algorithm

Worst Case Input

$$r_{\mathsf{ON}} = \max_{\sigma} \frac{v_{\mathsf{ON}}(\sigma)}{v_{\mathsf{OPT}}(\sigma)}$$

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With Stochastic Input

$$r_{\mathsf{ON}} = \frac{\mathbb{E}\{v_{\mathsf{ON}}(\sigma)\}}{\mathbb{E}\{v_{\mathsf{OPT}}(\sigma)\}}$$

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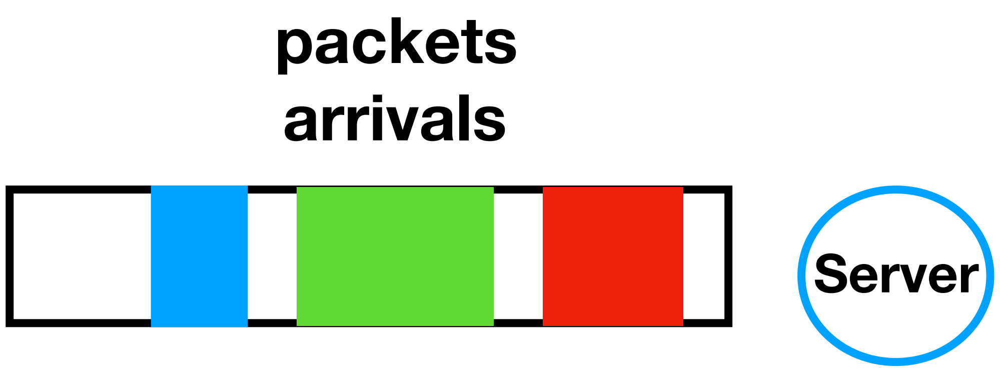
Goal online algorithm with least **CR**

Prior Work

What do we know !

What do we know !

Single server



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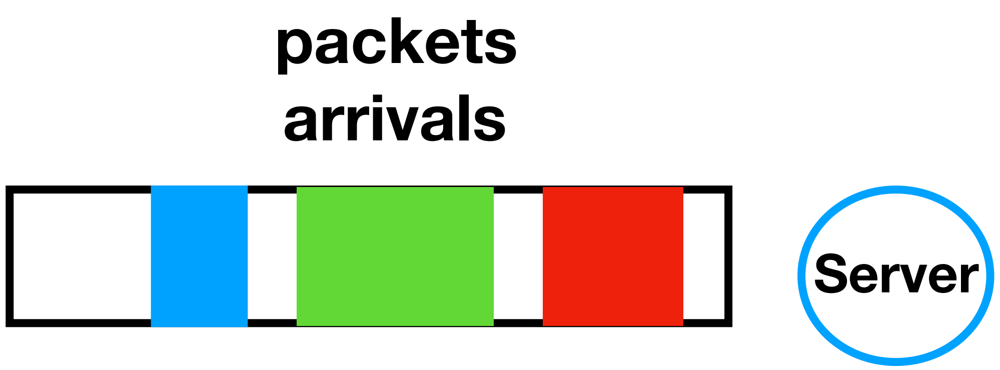

Opt Scheduling is SRPT

Optimal Speed choice

$$s(t) = P^{-1}(n(t))$$

$$P(s(t)) = n(t)$$

Best CR = 2

[Bansal et al 2009, Andrew et al 2010]

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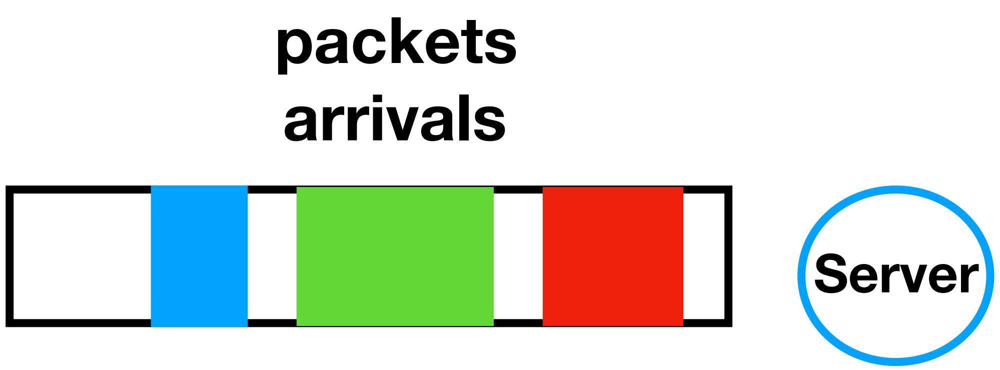

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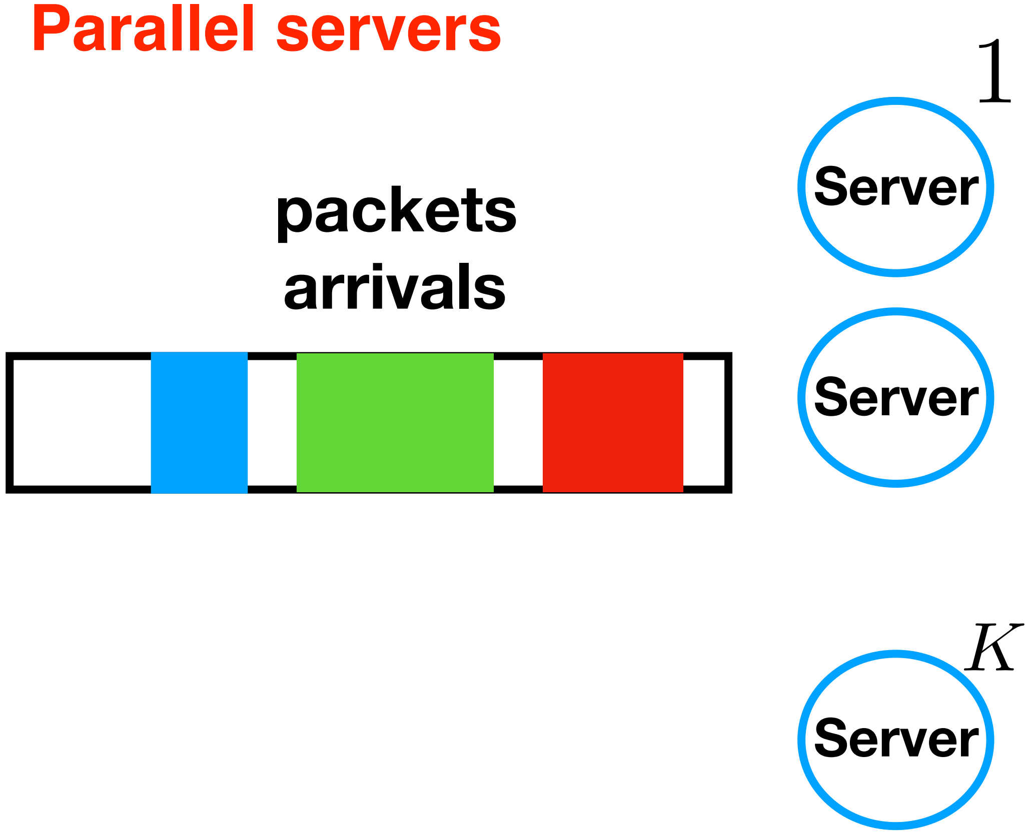
Parallel servers



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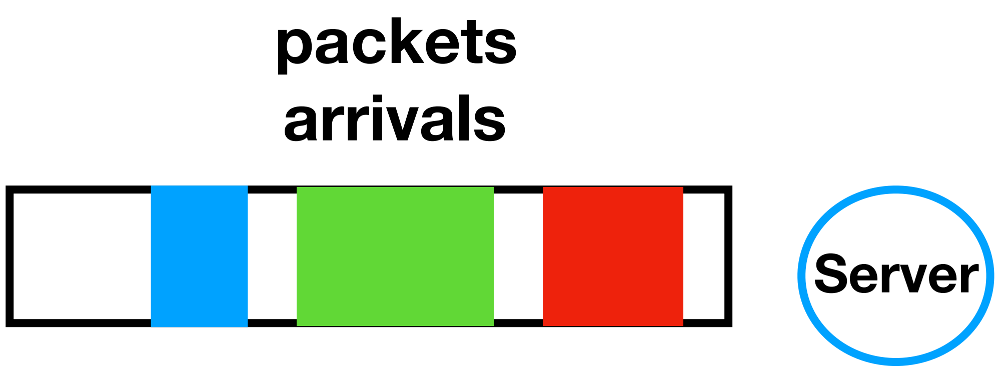

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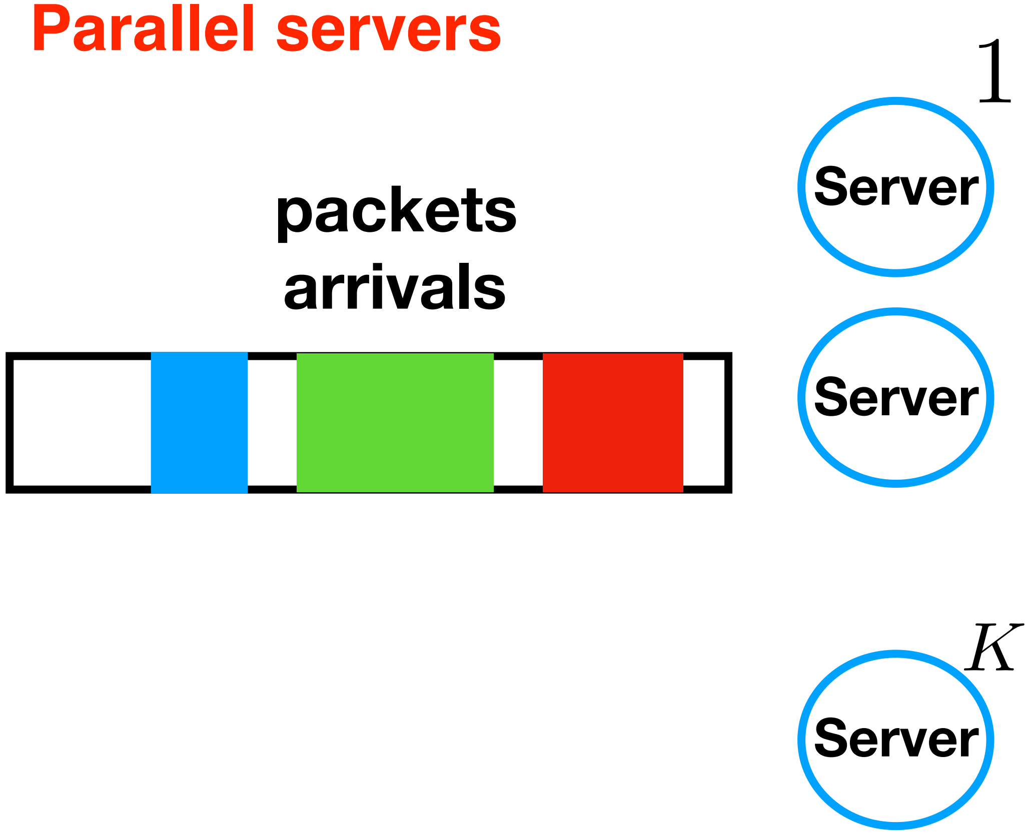

Opt Scheduling is unknown

Assign a new job to server $i^\star$

where $i^\star = \arg\max f_i(\text{current load} + \text{new job})$

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$O(\alpha)$ *[Gupta et al, 2010]*

2α *[Devanur et al, 2017]*

What do we know !

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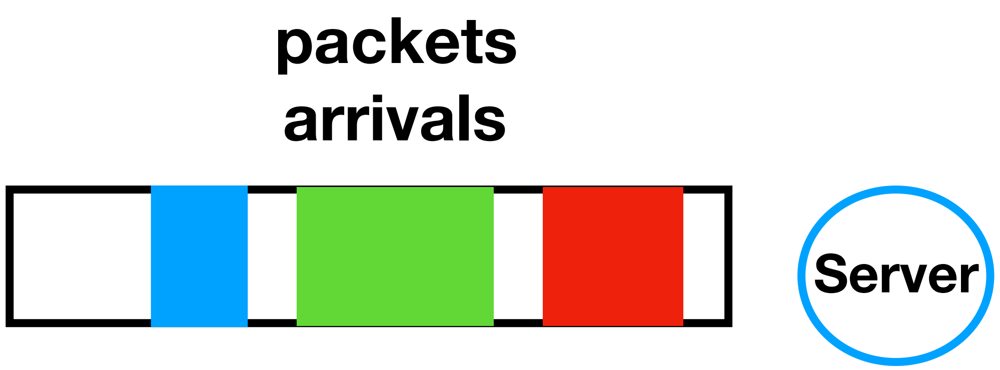

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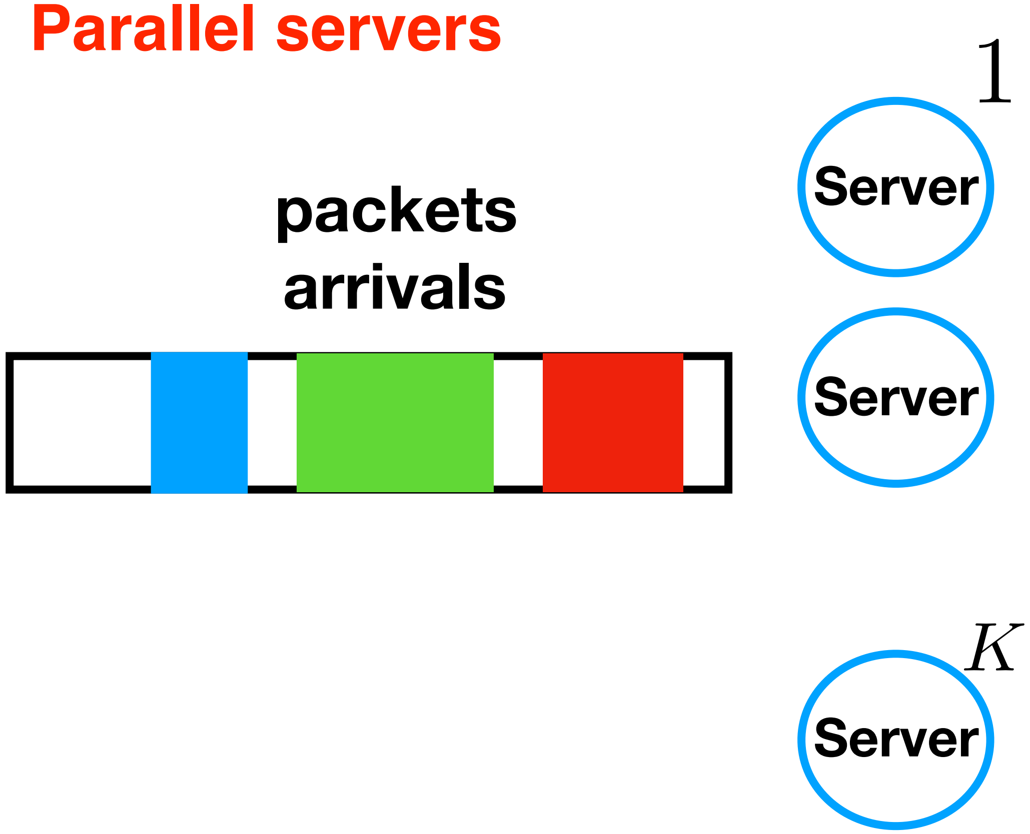

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Multi-Server SRPT

4.2^α *[**V.**, Nair, 2020]*

What do we know for Networks

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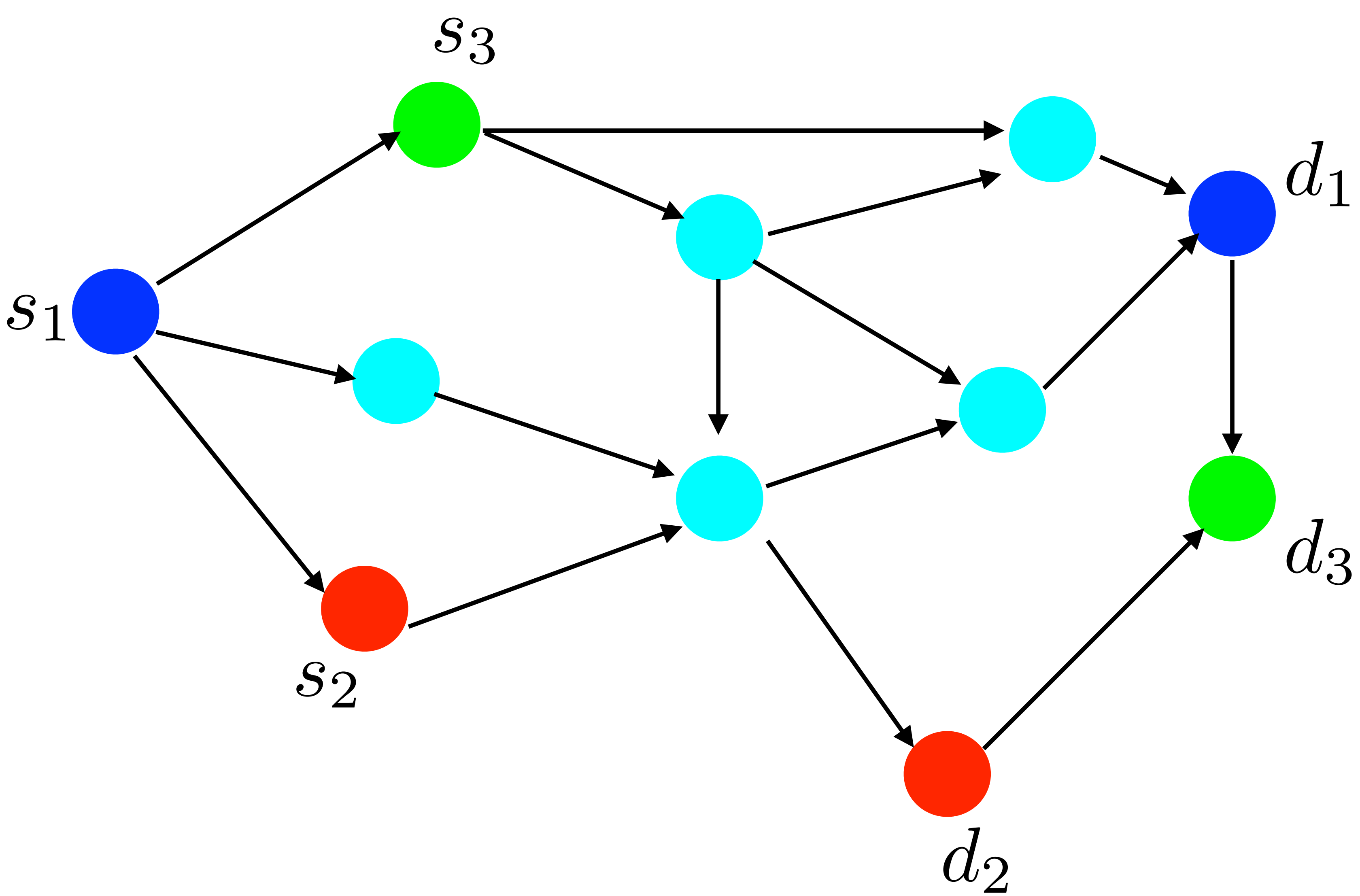

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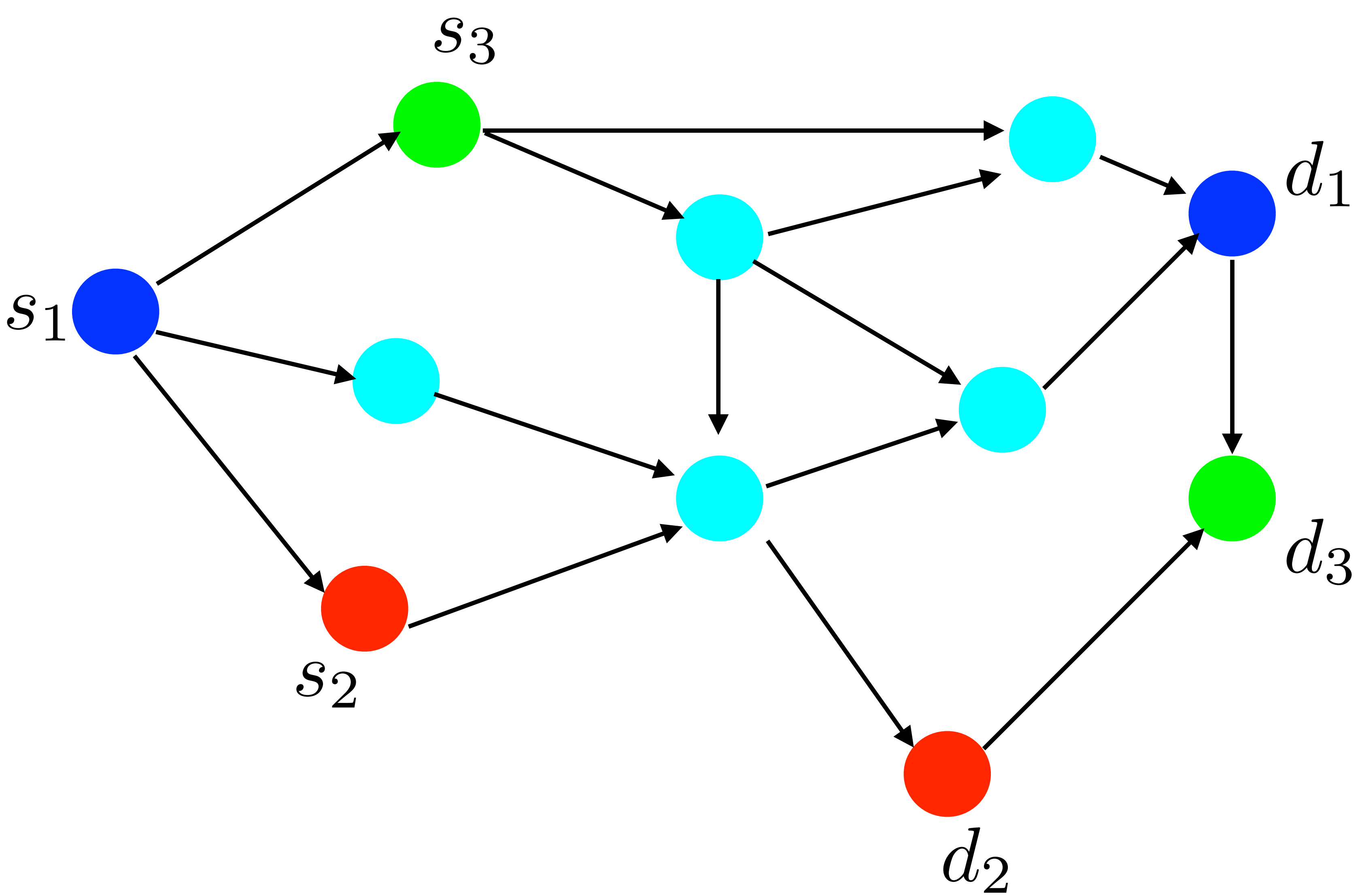

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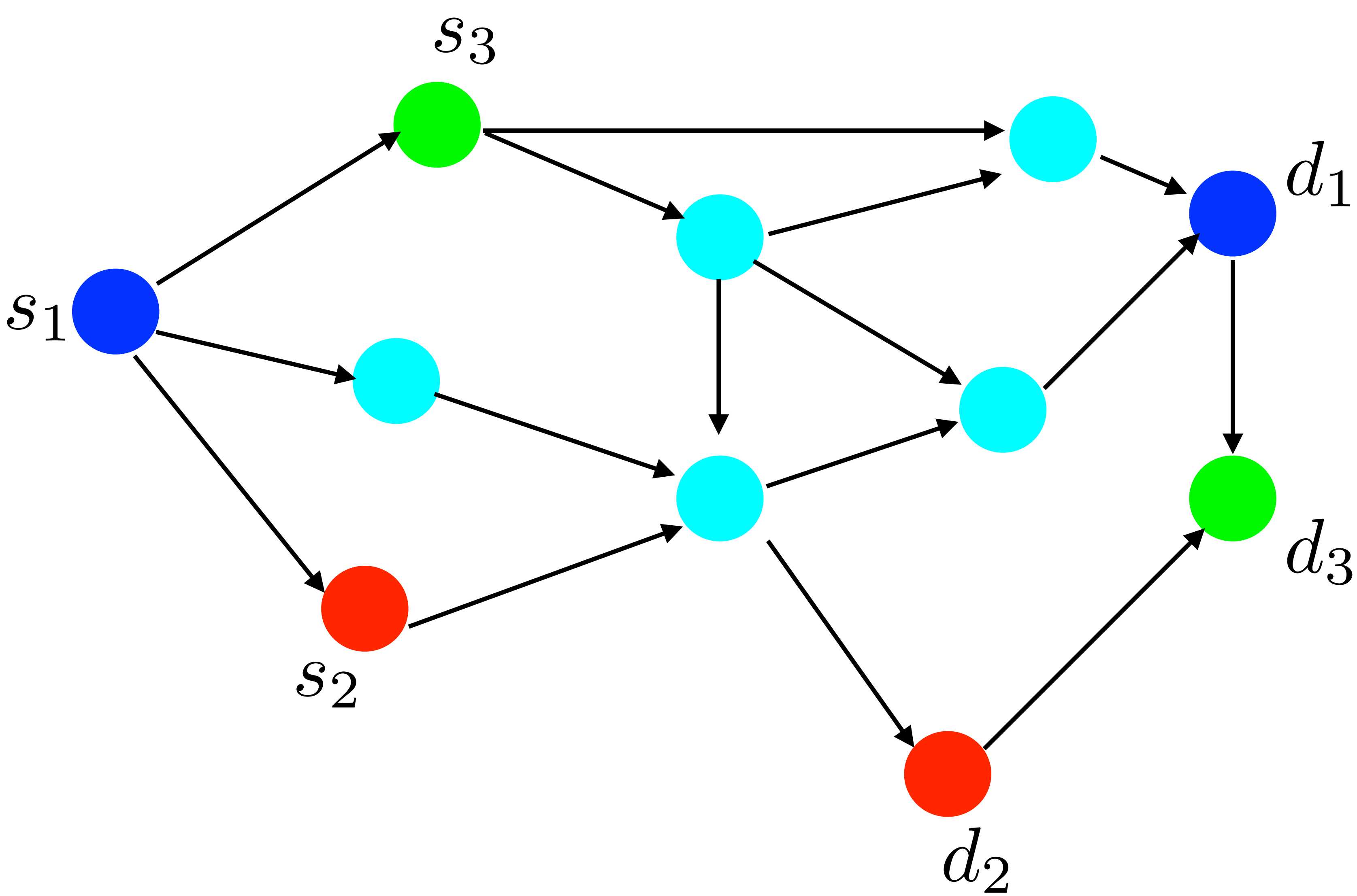
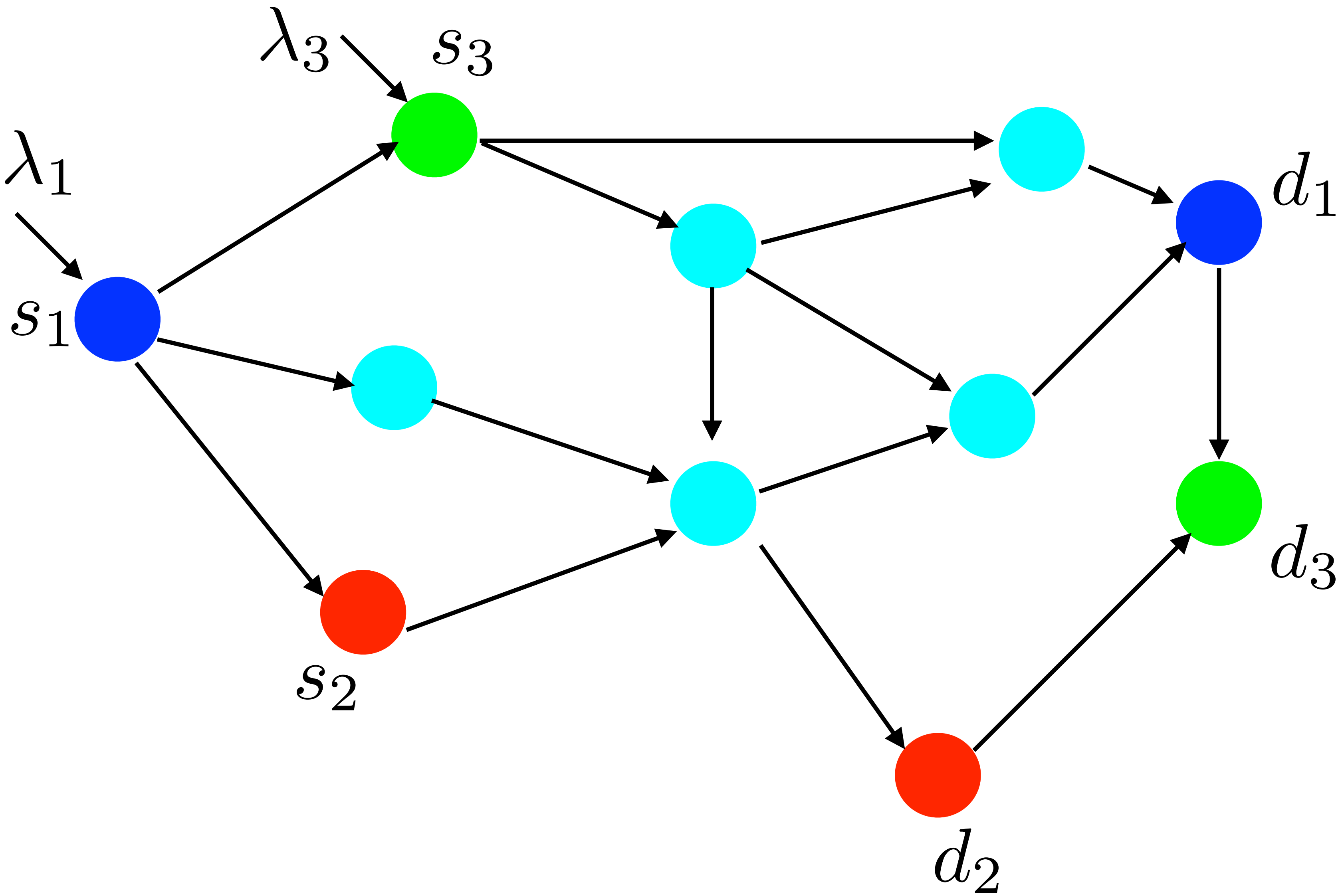


Not much !

Large body on Throughput Optimality

Stochastic Case

General Network - Stochastic Case



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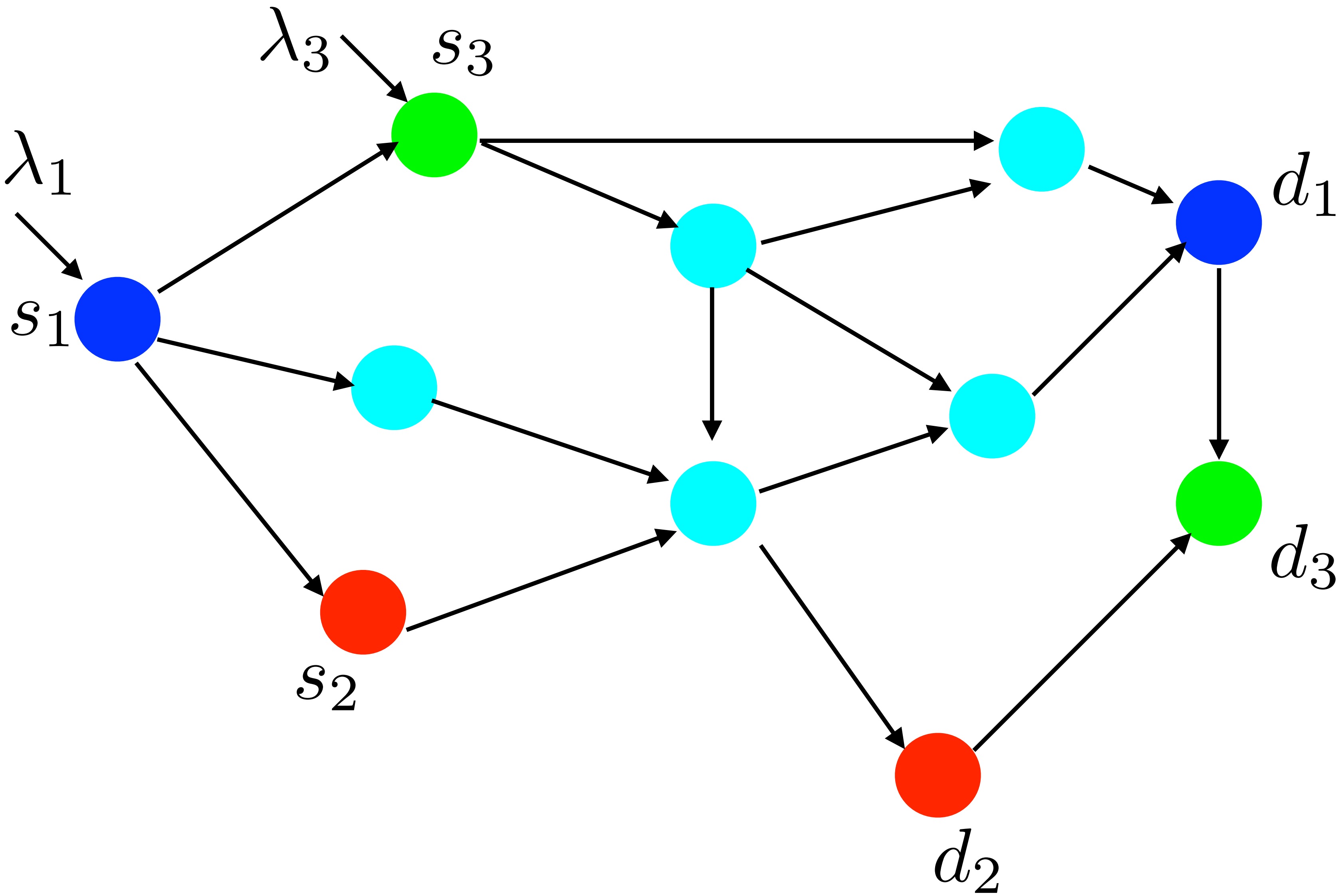

Poisson Arrivals with rate λ_i at source s_i

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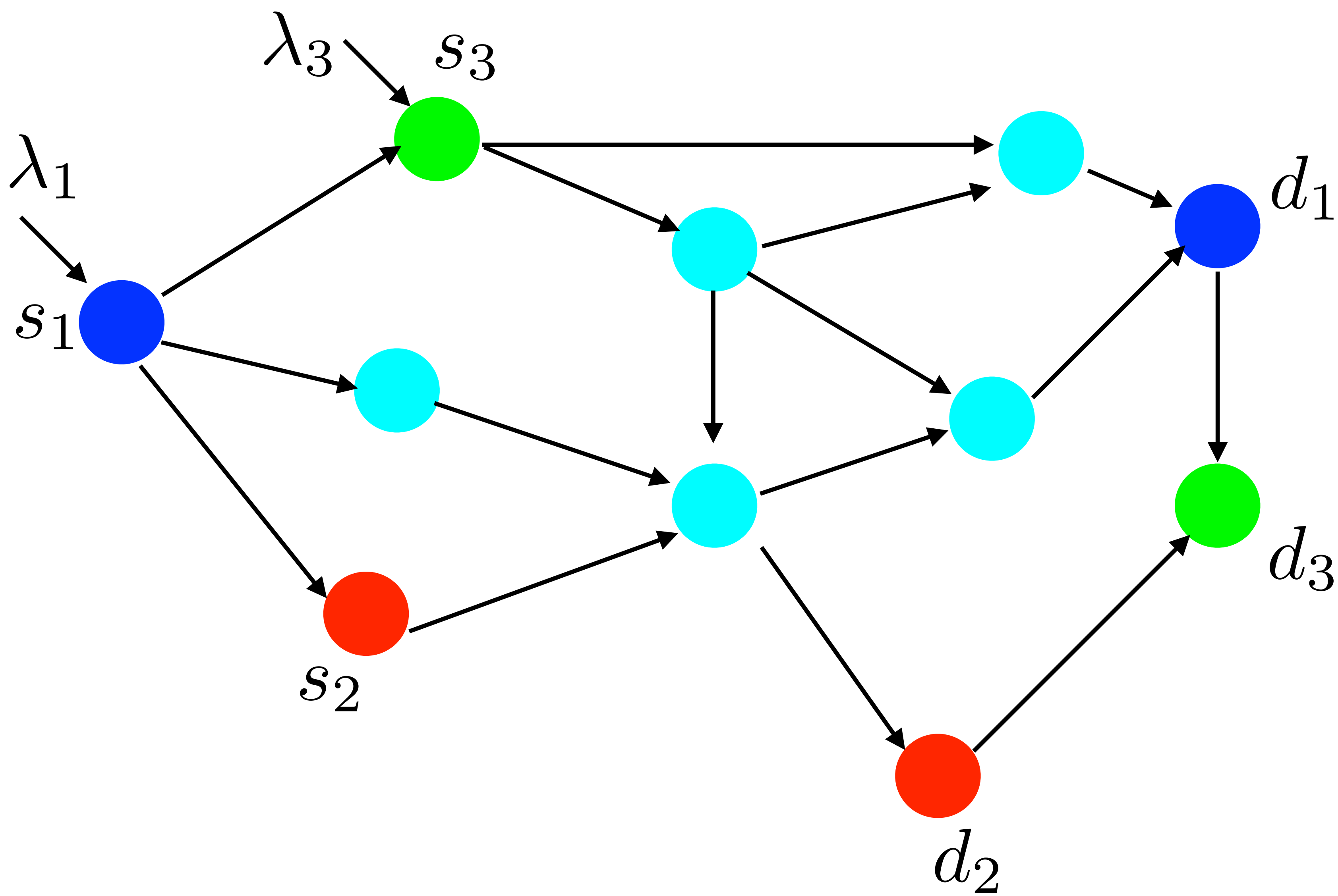

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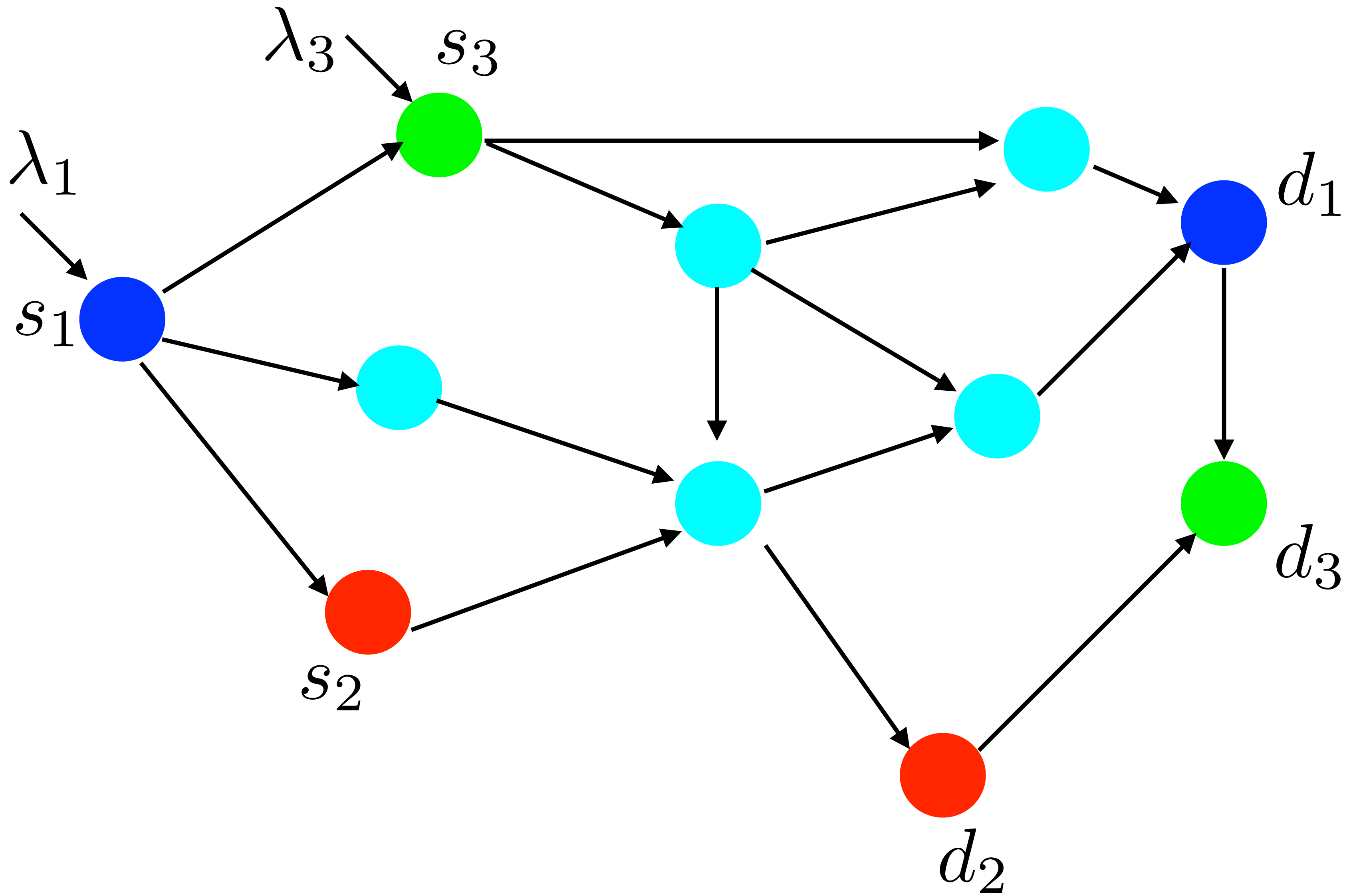
New Obj:

$$\min \mathbf{E}\left\{\sum_{i=1}^{\#\text{sources}} n_i(t)\right\} + \mathbf{E}\left\{\sum_{v\in V} P(s(t))\right\}$$

Flow time Energy

General Network - Stochastic Case



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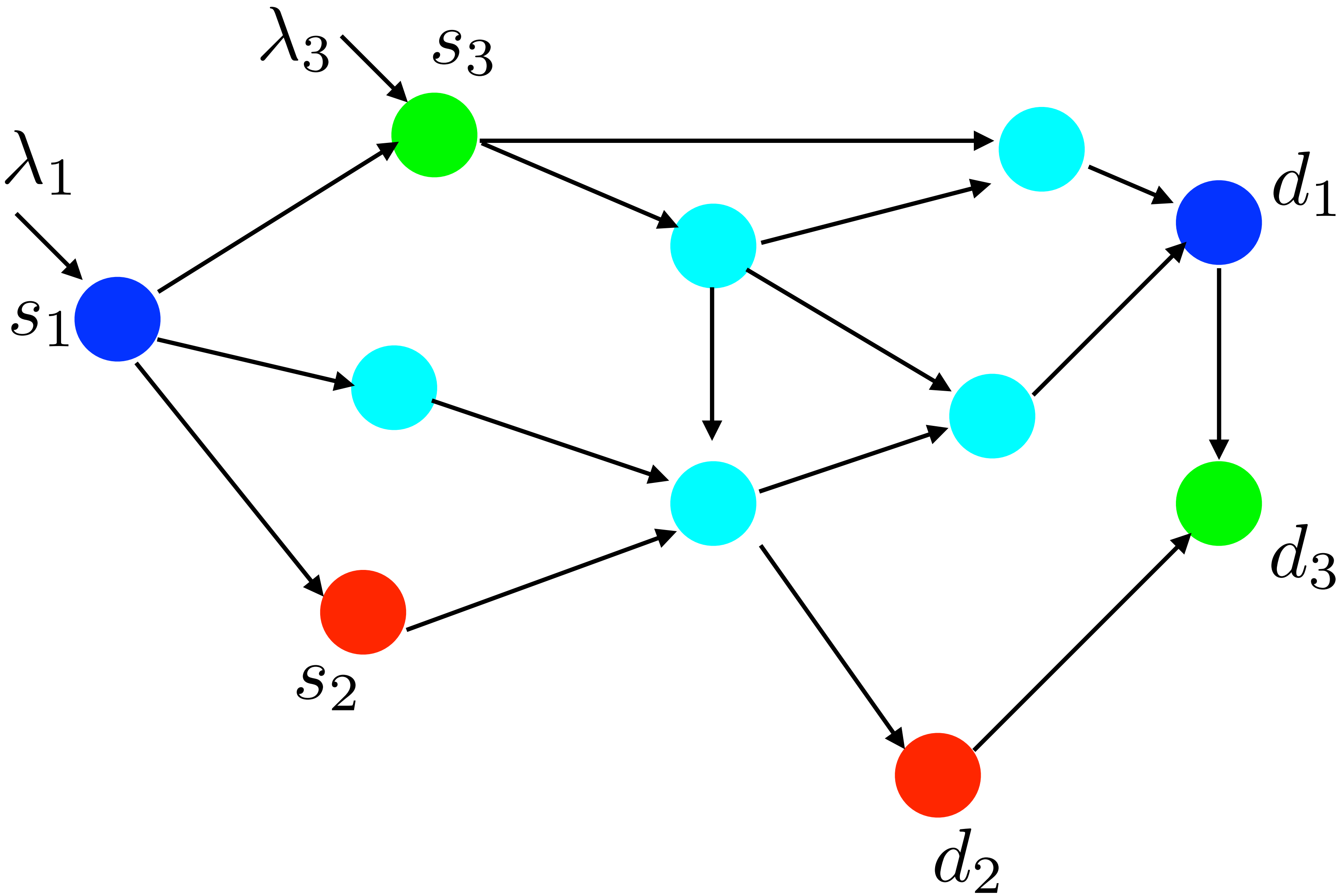
Flow time — Energy

Decisions

General Network - Stochastic Case



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Flow time

Energy

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Routing

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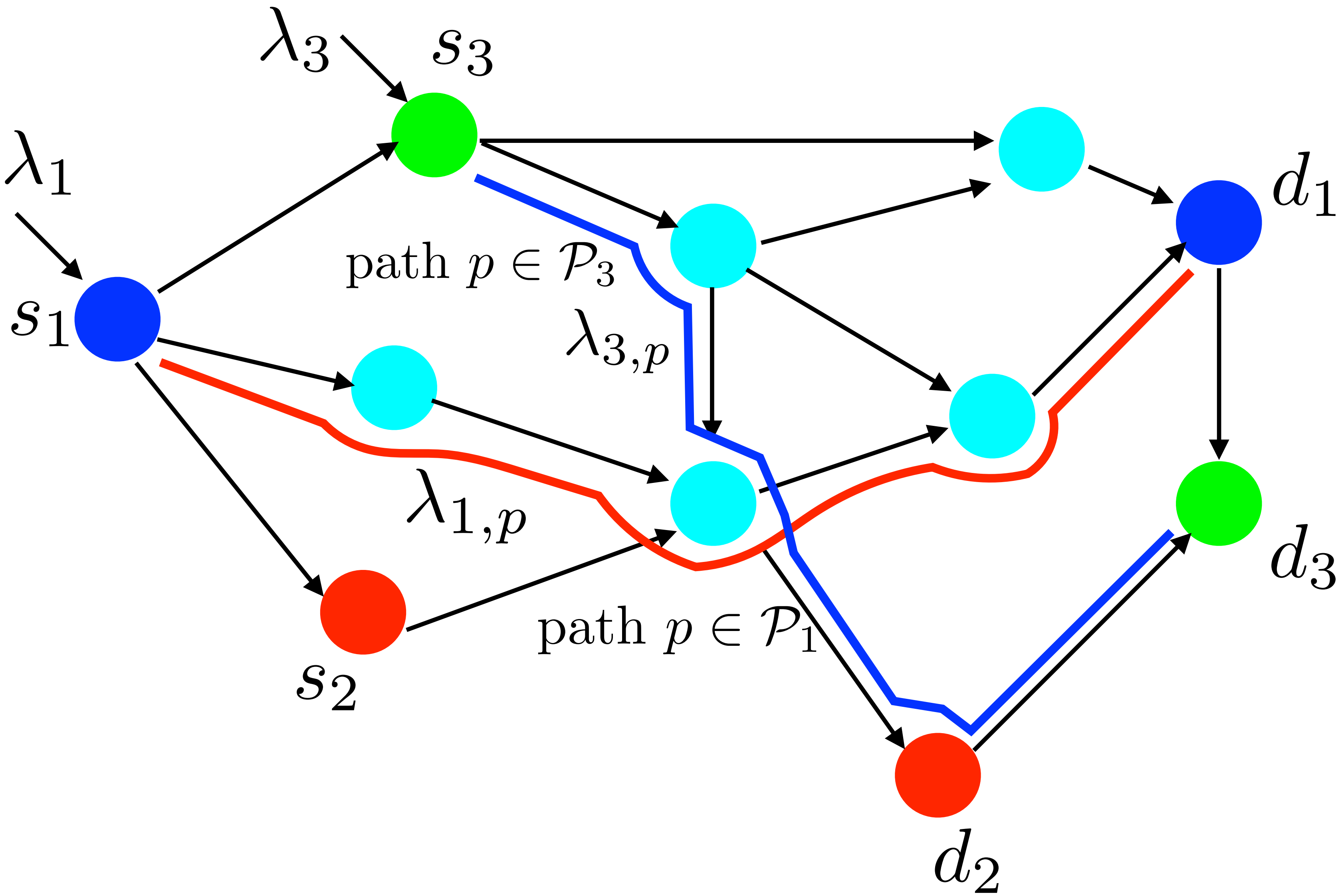

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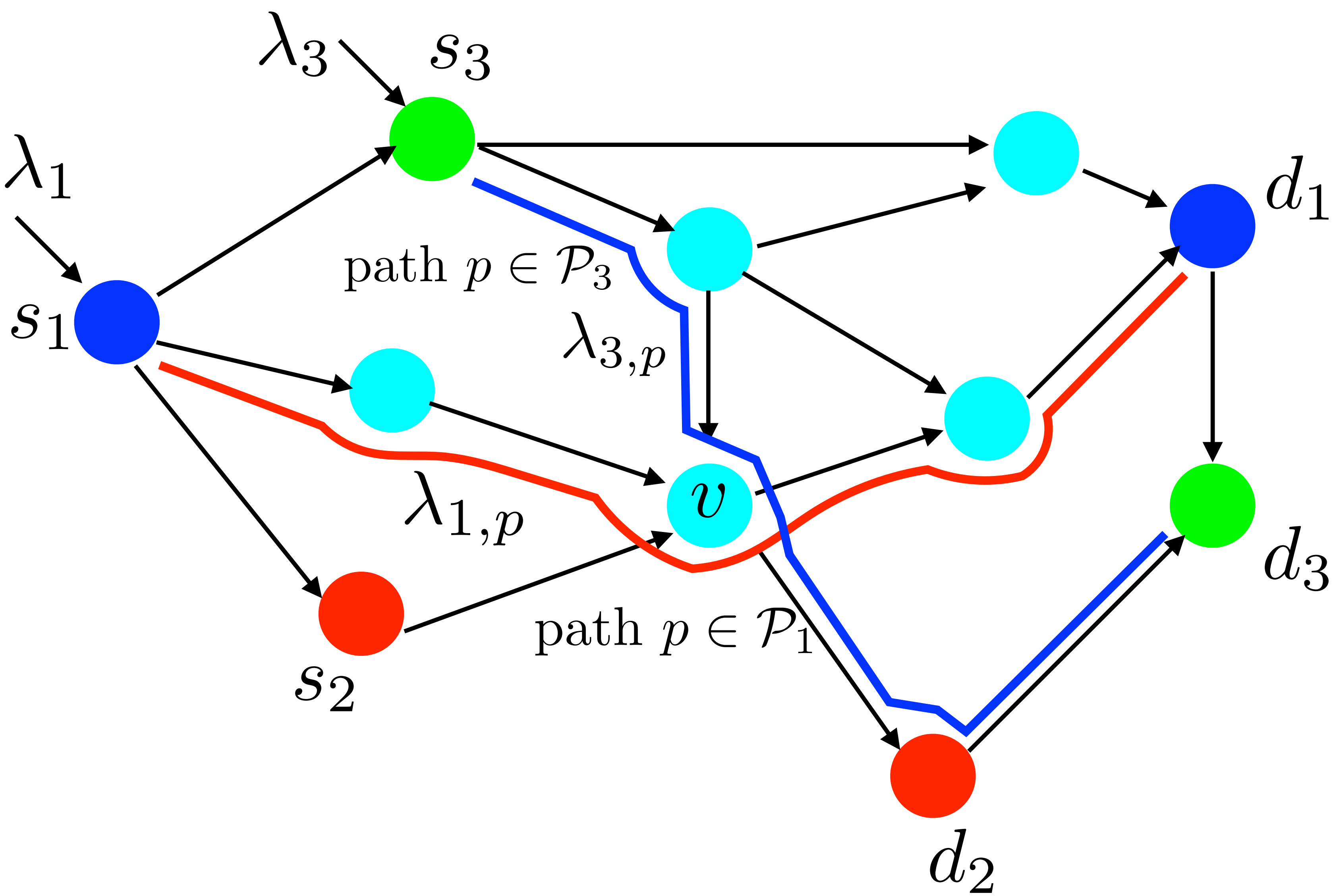

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Flow time — Energy

Decisions — **Routing** — **Speed** v

Lower Bounding the Cost of OPT

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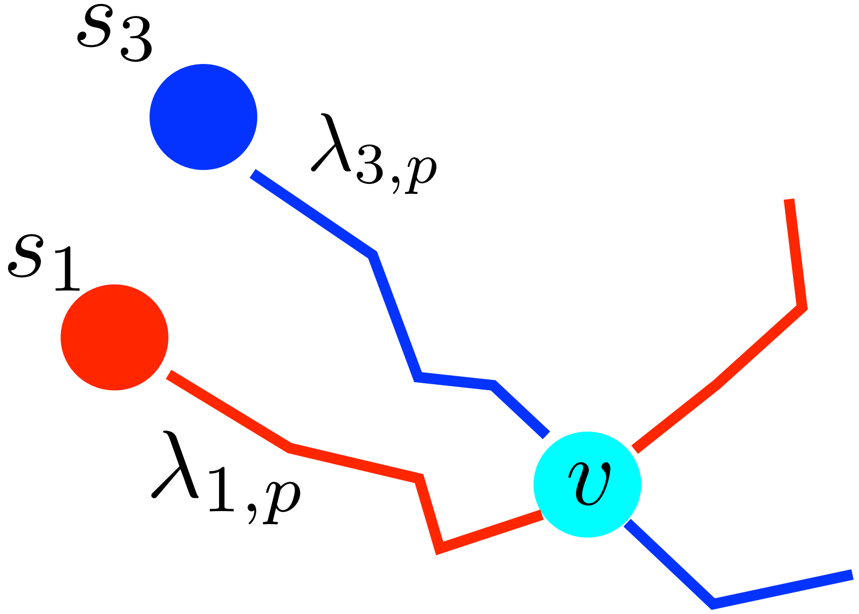
$\lambda_{i,p}$ **Any feasible flow on path p for the i^th^ S-D pair**

Lower Bounding the Cost of OPT



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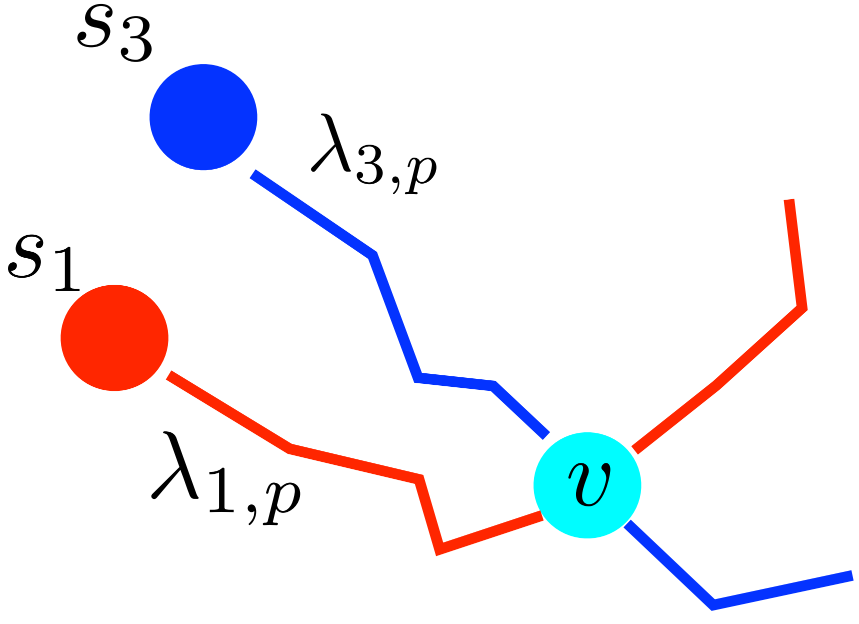
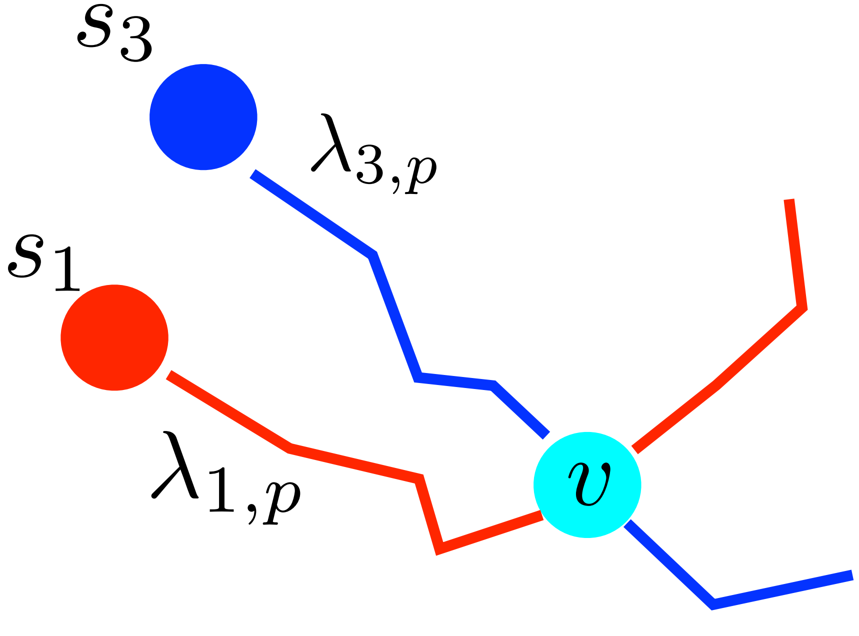


$\lambda_{i,p}$ **Any feasible flow on path p for the i^th S-D pair**

$$\lambda_v = \sum_{i=1}^{\#\text{sources}} \sum_{p \in \mathcal{P}_i:\ v \in p} \lambda_{i,p}$$

Total Traffic passing Through node v

Lower Bounding the Cost of OPT



Exp. Energy Cost for node v

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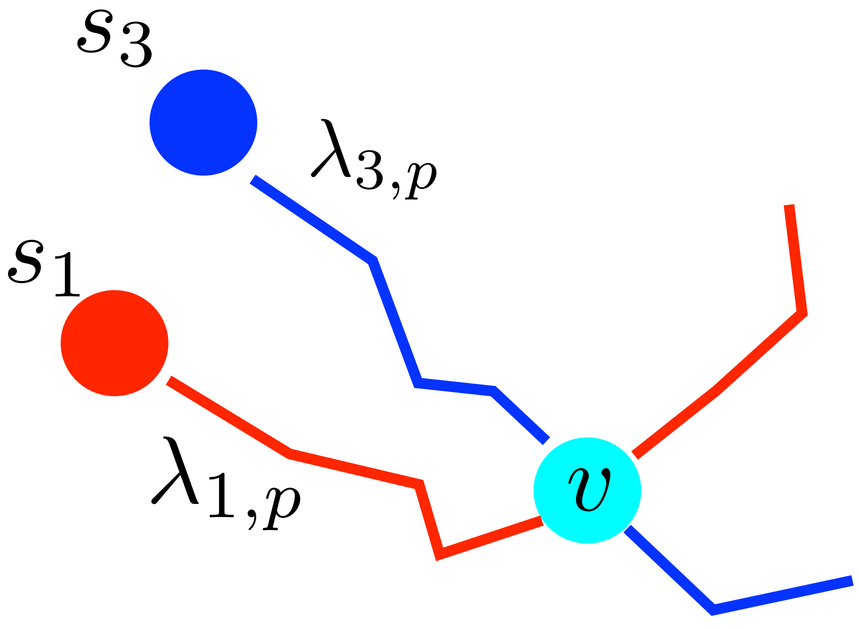

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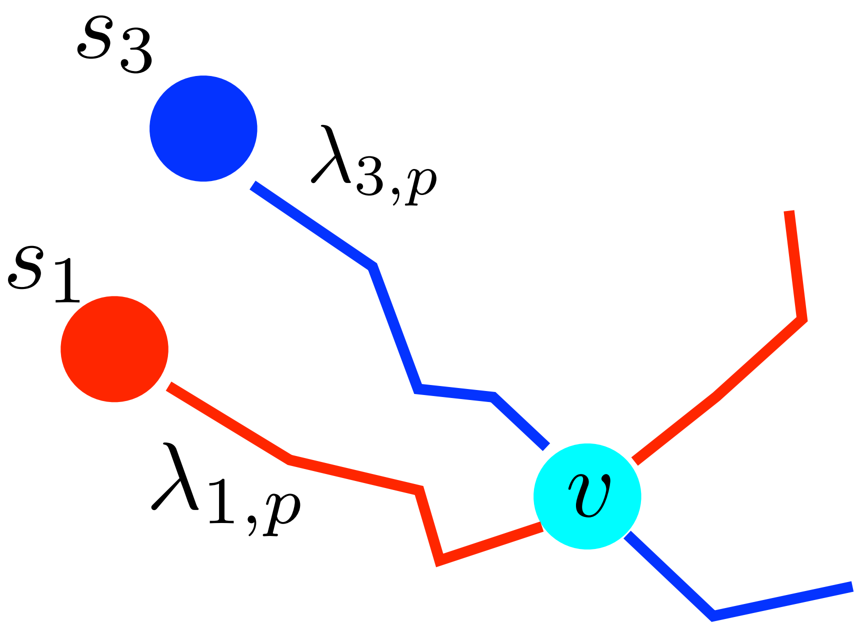
Total Traffic passing Through node v

$$\mathbb{E}\{P(s_v)\} \overset{Jensen's}{\geq} P(\mathbb{E}\{s_v\}) \overset{Stability}{\geq} P(\lambda_v)$$

Lower Bounding the Cost of OPT



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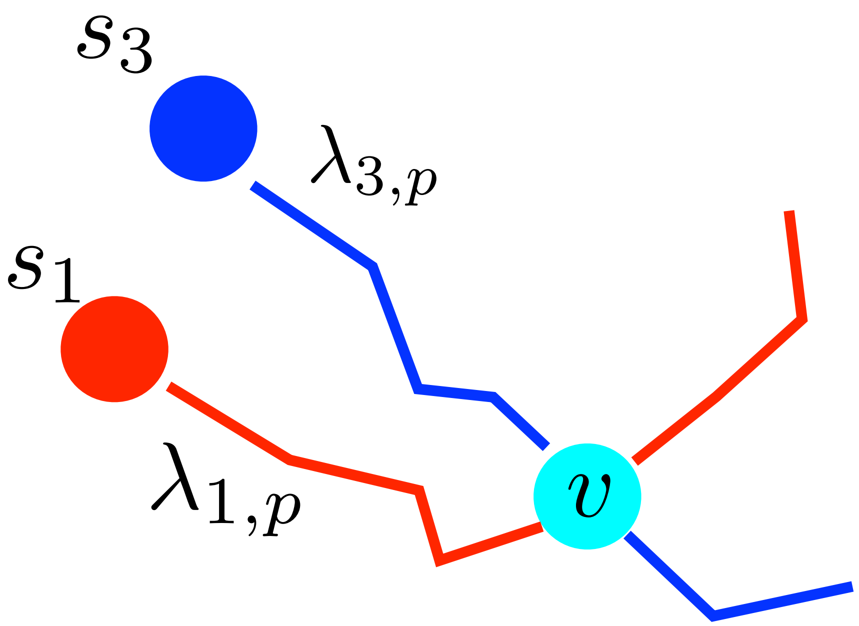

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Solve Convex Program : Total Power

$$\text{min.} \quad \sum_{v \in \mathcal{V}} \left(\lambda_v \right)^{\alpha}$$

$$\text{s.t.} \quad \sum_{p \in \mathcal{P}_i} \lambda_{i,p} = \lambda_i \quad \forall \quad \text{flows } i$$

$$\lambda_{i,p} \geq 0 \quad \forall \quad \text{flows } i,\ p \in \mathcal{P}_i$$

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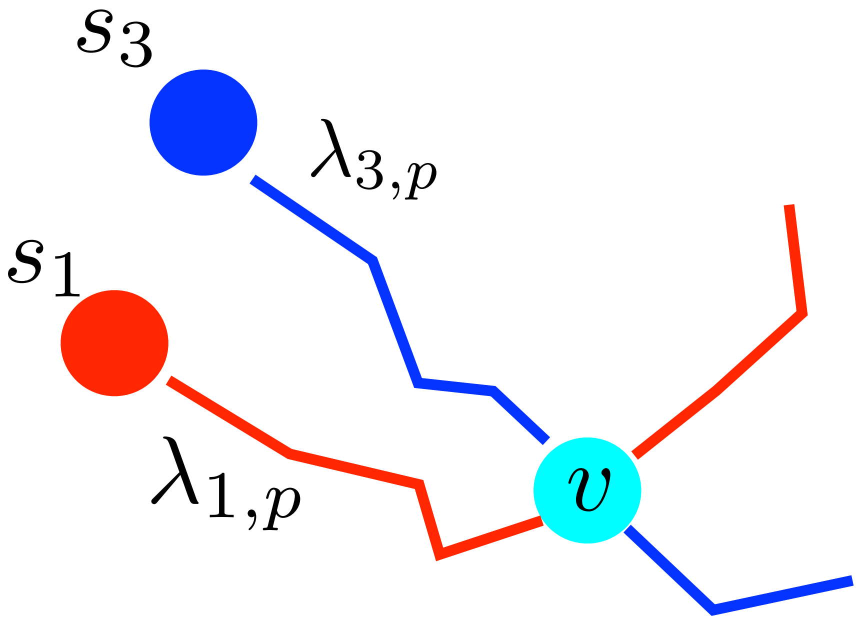
Lower Bound on the Energy Cost Of the Network for OPT

Lower Bounding the Cost of OPT



Exp. Energy Cost for node v

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Solve Convex Program : Total Power

$$\text{min.} \quad \sum_{v \in \mathcal{V}} \left(\sum_{i=1}^{\mathcal{D}} \sum_{p \in \mathcal{P}_i:\ v \in p} \lambda_{i,p} \right)^{\alpha}$$

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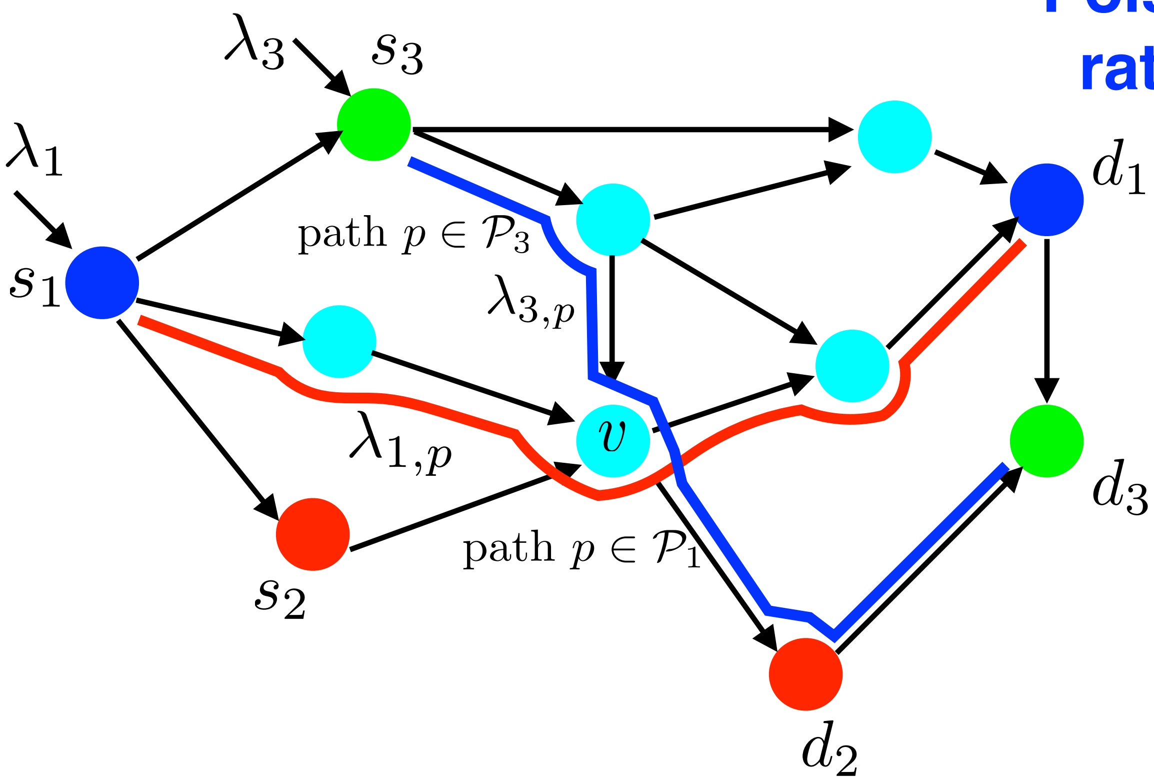
Lower Bound on the Energy Cost
Of the Network for OPT

Algorithm

Poisson Arrivals with rate λ_i at source s_i



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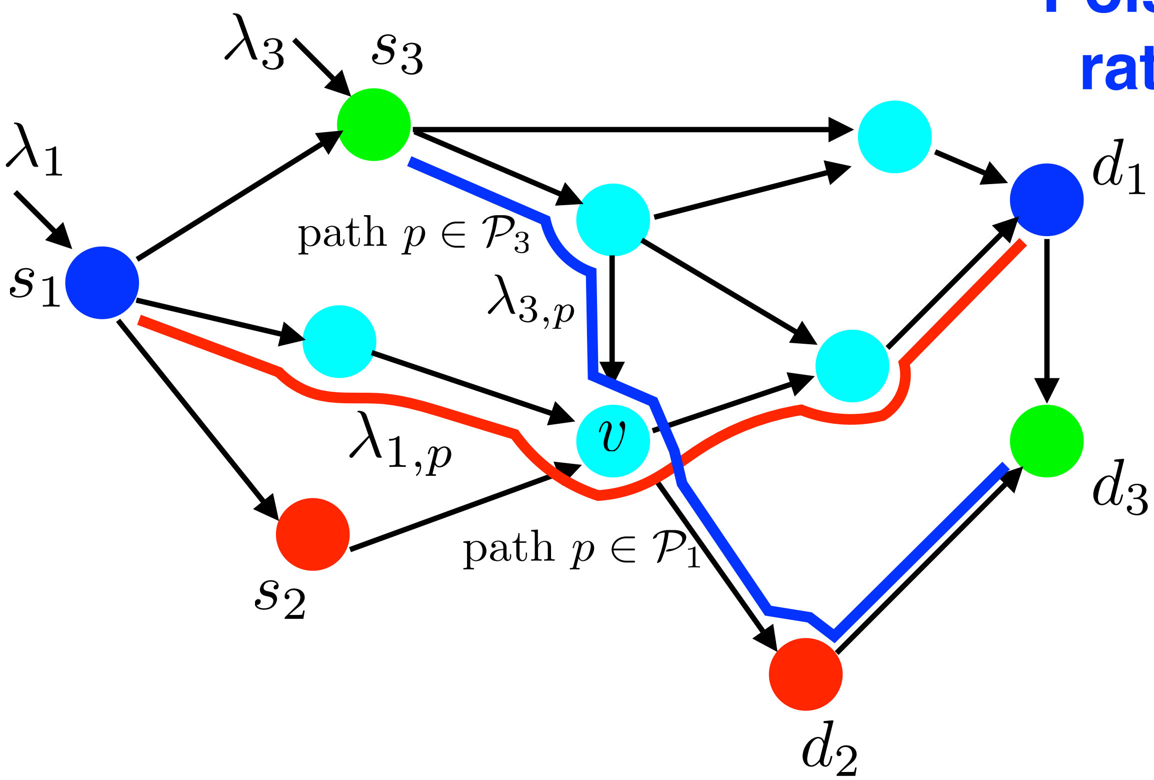

Poisson Arrivals with rate λ_i at source s_i

Algorithm

Solve Convex Program : Total Power

$$\text{min.} \quad \sum_{v \in \mathcal{V}} \left(\sum_{i=1}^{\mathcal{D}} \sum_{p \in \mathcal{P}_i :\ v \in p} \lambda_{i,p} \right)^{\alpha}$$

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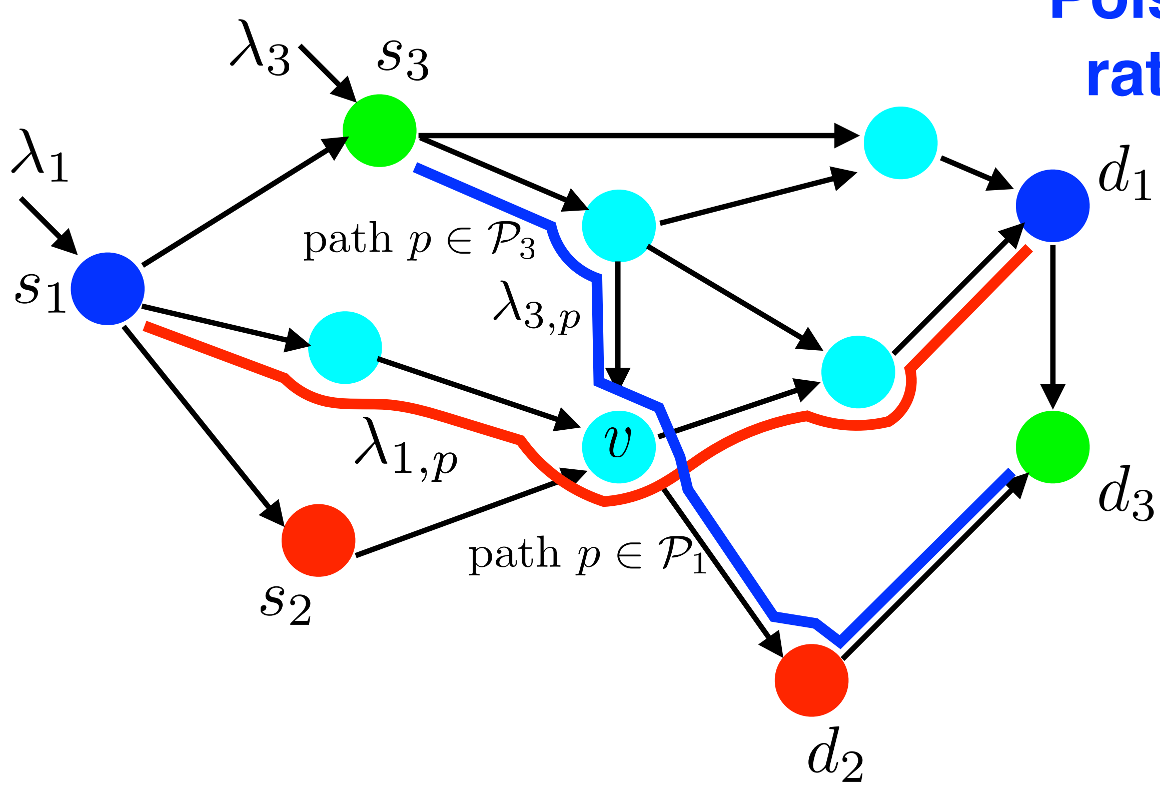


Poisson Arrivals with rate λ_i at source s_i

Algorithm

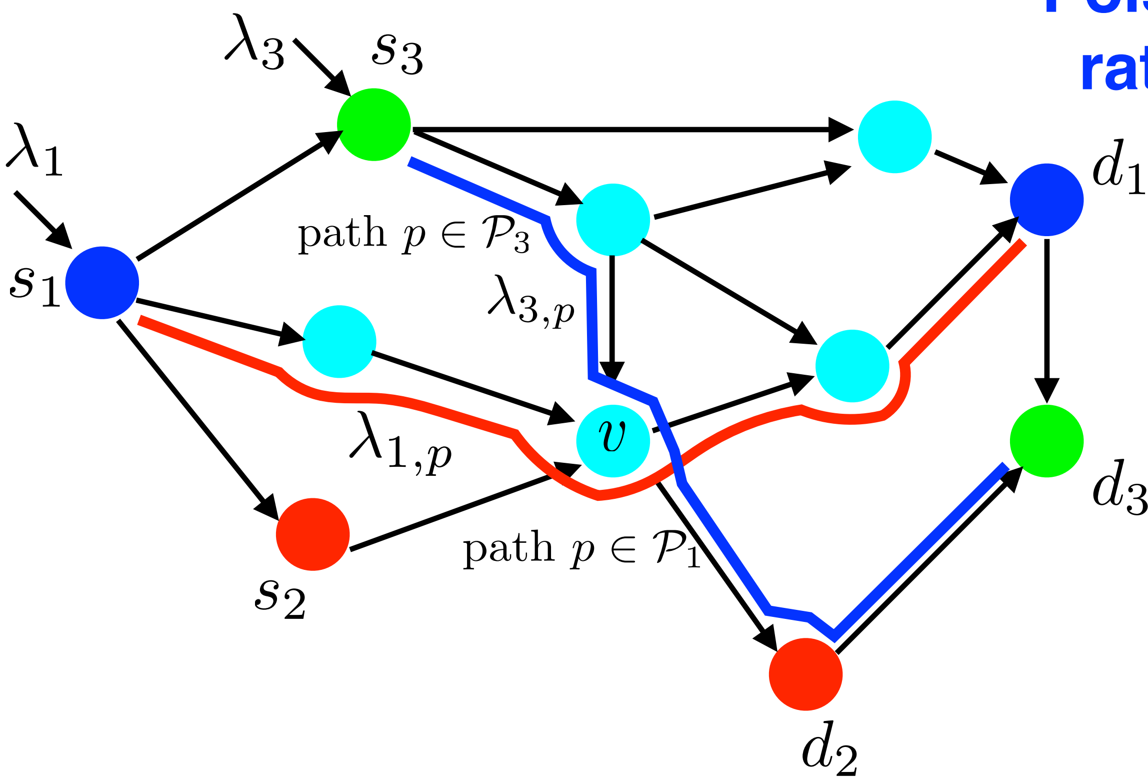
Routing : On path p of i^{th} S-D pair
Route $\lambda^{\star}_{i,p}$

Solve Convex Program : Total Power

$$\min. \quad \sum_{v \in \mathcal{V}} \left(\sum_{i=1}^{\mathcal{D}} \sum_{p \in \mathcal{P}_i:\ v \in p} \lambda_{i,p} \right)^{\alpha}$$

$$\text{s.t.} \quad \textstyle\sum_{p \in \mathcal{P}_i} \lambda_{i,p} = \lambda_i \quad \forall \quad \text{flows } i$$
$$\lambda_{i,p} \geq 0 \quad \forall \quad \text{flows } i,\ p \in \mathcal{P}_i$$

Algorithm



Poisson Arrivals with rate λ_i at source s_i

Algorithm

Routing : On path p of i^{th} S-D pair Route $\lambda^{\star}_{i,p}$

Speed : Node *v* uses speed

Solve Convex Program : Total Power

$$\text{min.} \quad \sum_{v \in \mathcal{V}} \left(\sum_{i=1}^{\mathcal{D}} \sum_{p \in \mathcal{P}_i :\ v \in p} \lambda_{i,p} \right)^{\alpha}$$

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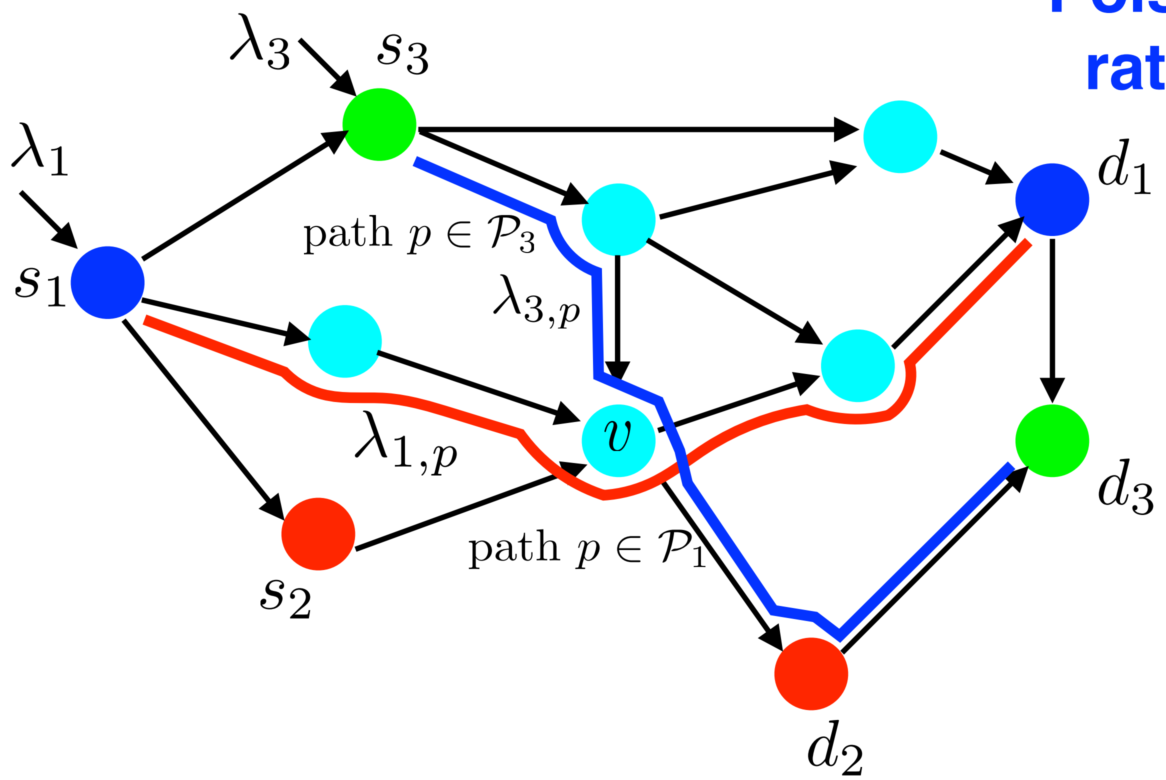

Poisson Arrivals with rate λ_i at source s_i

Algorithm

Routing : On path p of i^{th} S-D pair
Route $\lambda^\star_{i,p}$

Speed : Node *v* uses speed

$$s_v = \lambda^\star_v + \min\left\{1, \frac{1}{2(\alpha-1)}\right\}$$

A bit more than arrival rate

Solve Convex Program : Total Power

$$\text{min.} \quad \sum_{v\in\mathcal{V}} \left(\sum_{i=1}^{\mathcal{D}} \sum_{p\in\mathcal{P}_i:\ v\in p} \lambda_{i,p} \right)^{\alpha}$$

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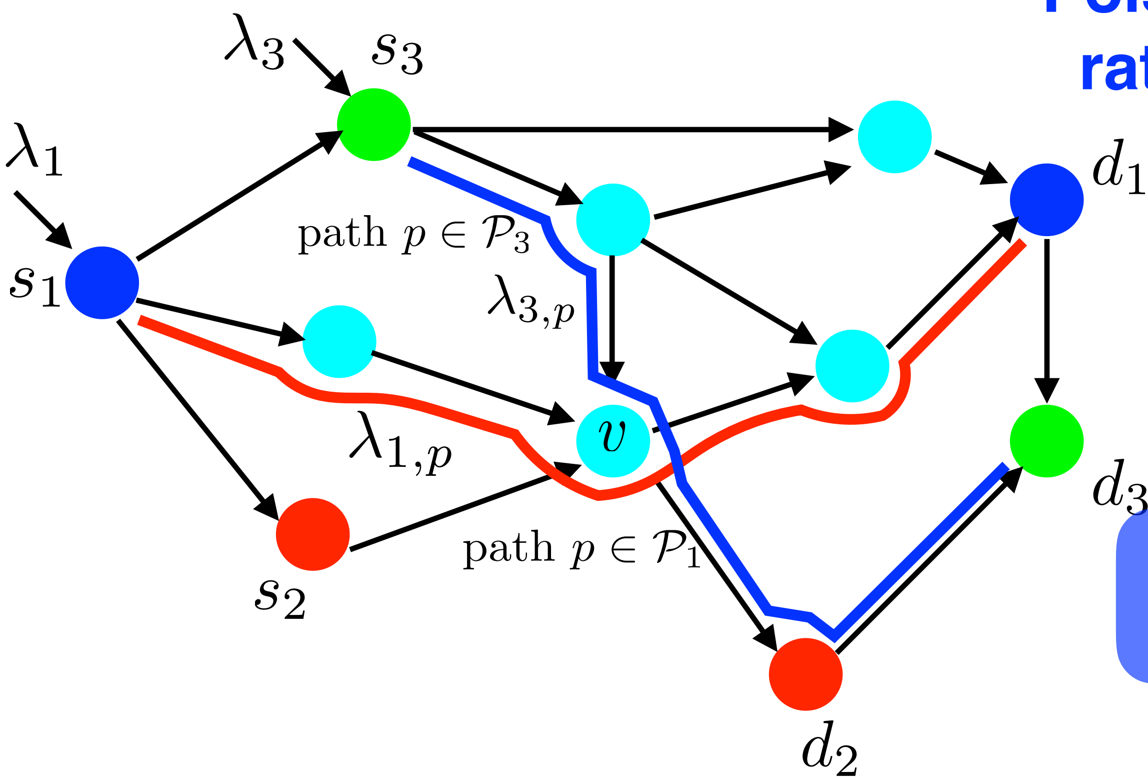

Poisson Arrivals with rate λ_i at source s_i

$$P(s) = s^{\alpha}$$

Algorithm

Routing : On path p of i^{th} S-D pair
Route $\lambda^{\star}_{i,p}$

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A bit more than arrival rate

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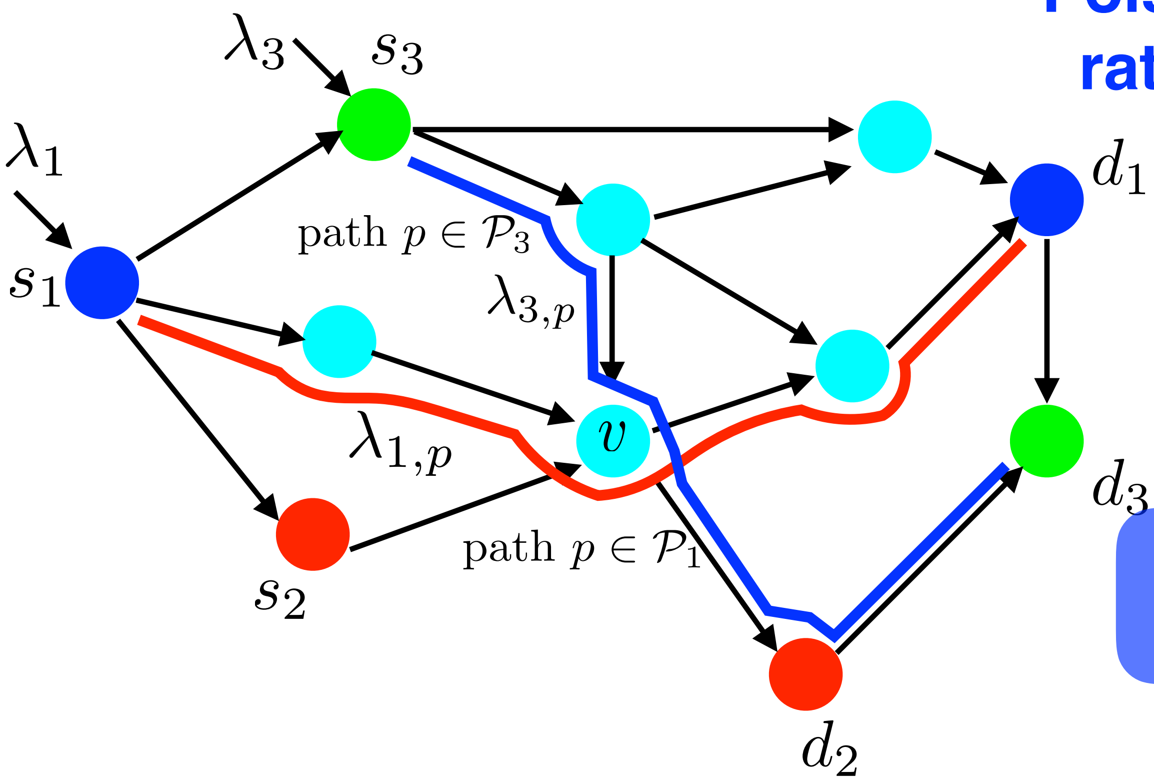

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A bit more than arrival rate

CR = $\theta_{\alpha} \cdot \theta_{\text{network}}$

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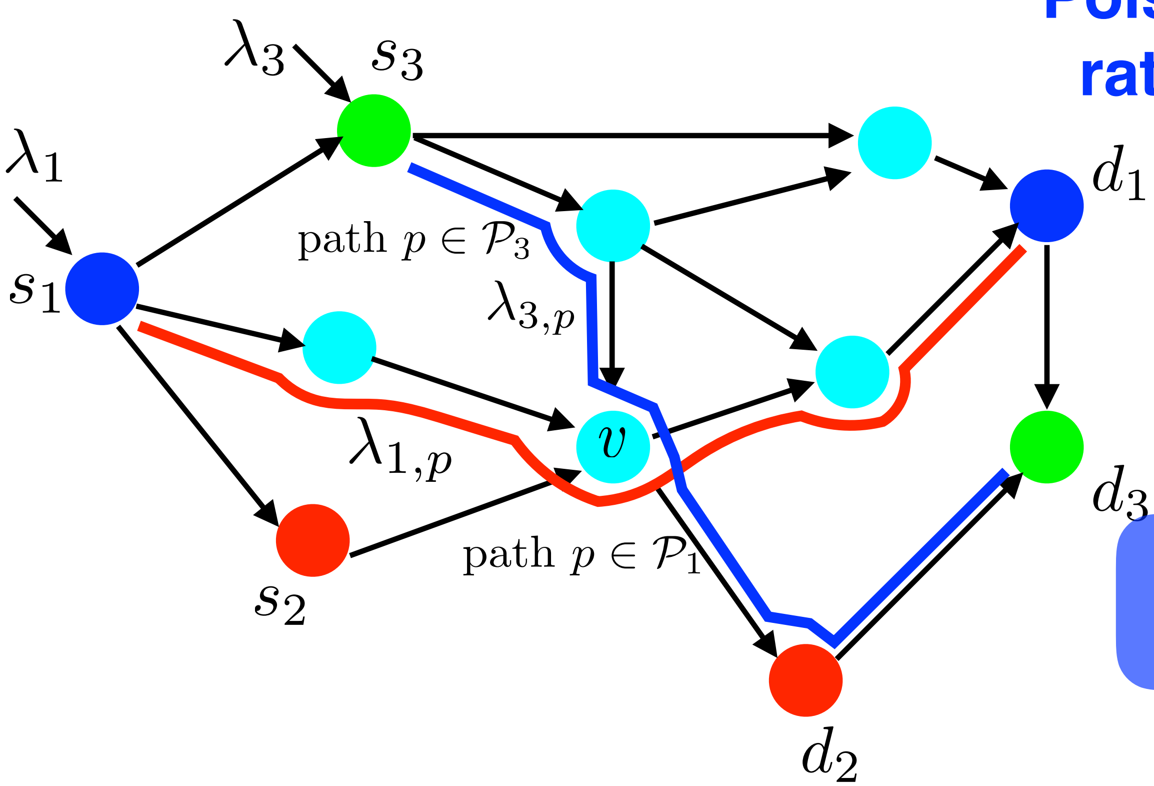

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A bit more than arrival rate

CR = $\theta_\alpha . \ \theta_{\text{network}}$

$$\theta_\alpha = 3 + \max\{1, 2\alpha - 2\}$$

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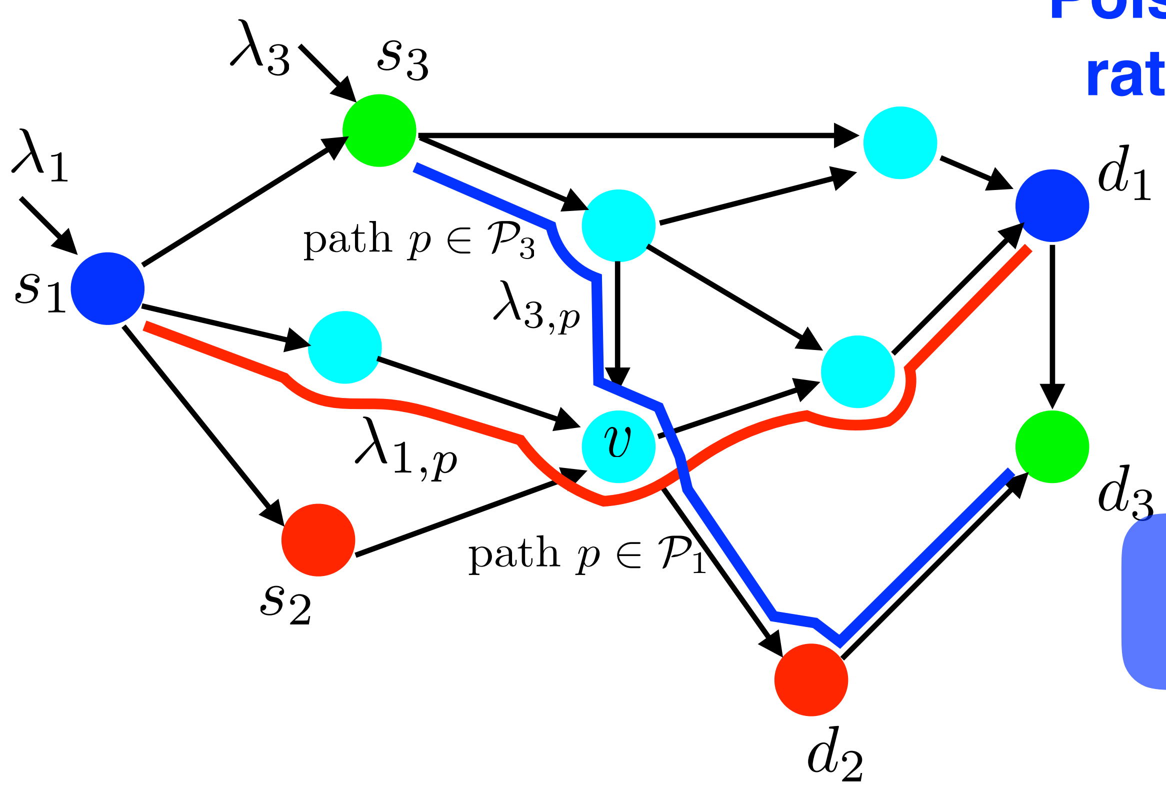

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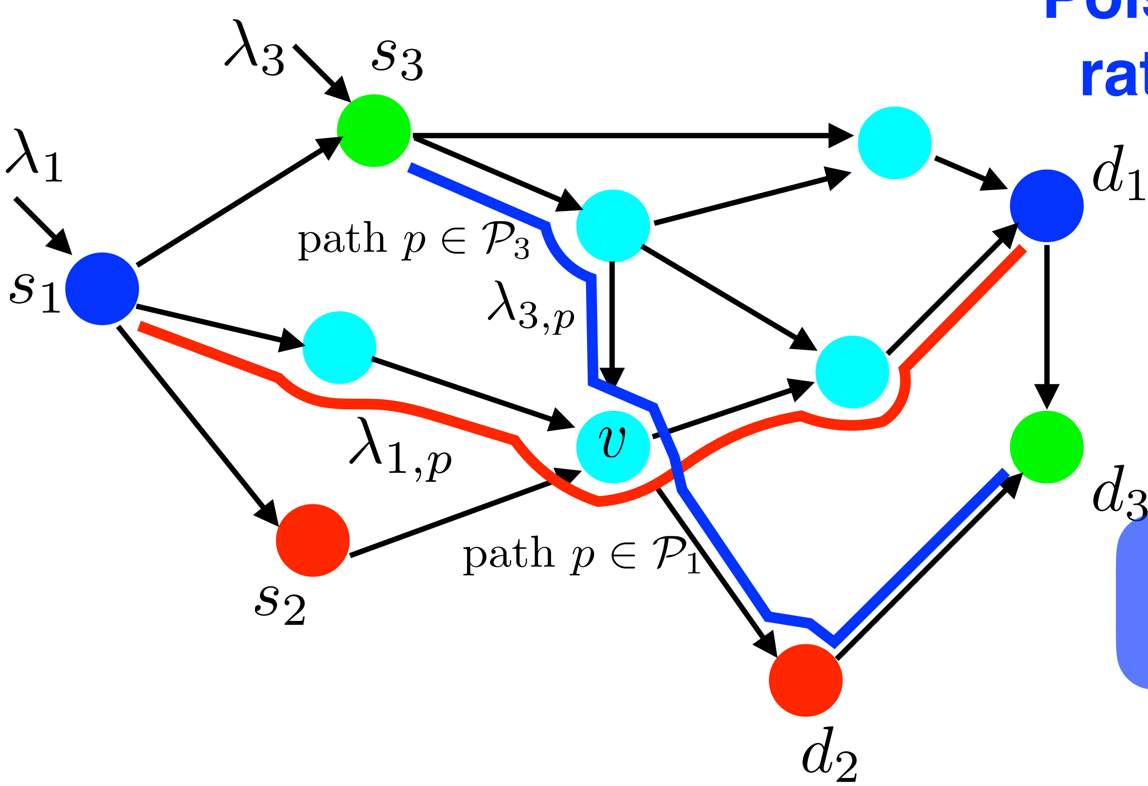
i^th S-D pair

Algorithm



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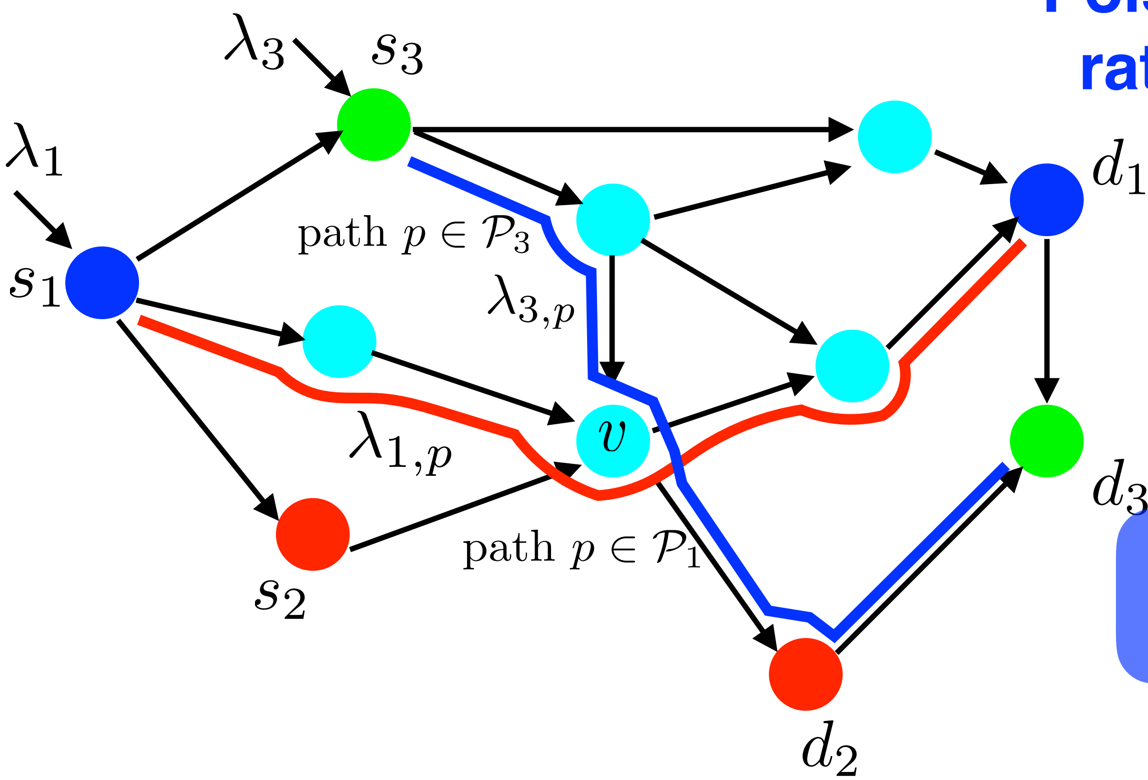
i^{th} S-D pair

$L_p = \text{Length of path p}$

Algorithm



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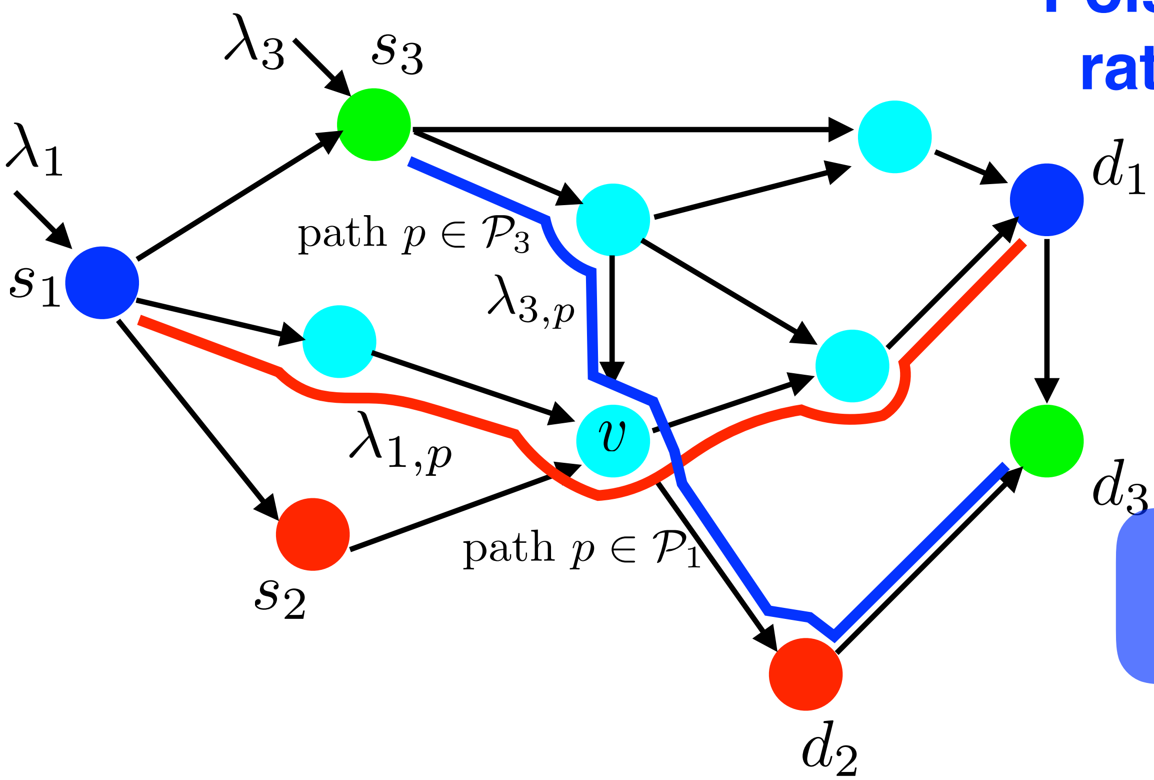

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i^th S-D pair

L_p = Length of path p

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$$\theta_i = \frac{1}{\lambda_i} \sum_{\text{paths } p \text{ from } s_i \to d_i} \lambda^{\star}_{i,p} \frac{L_p}{L_{\min}(i)}$$

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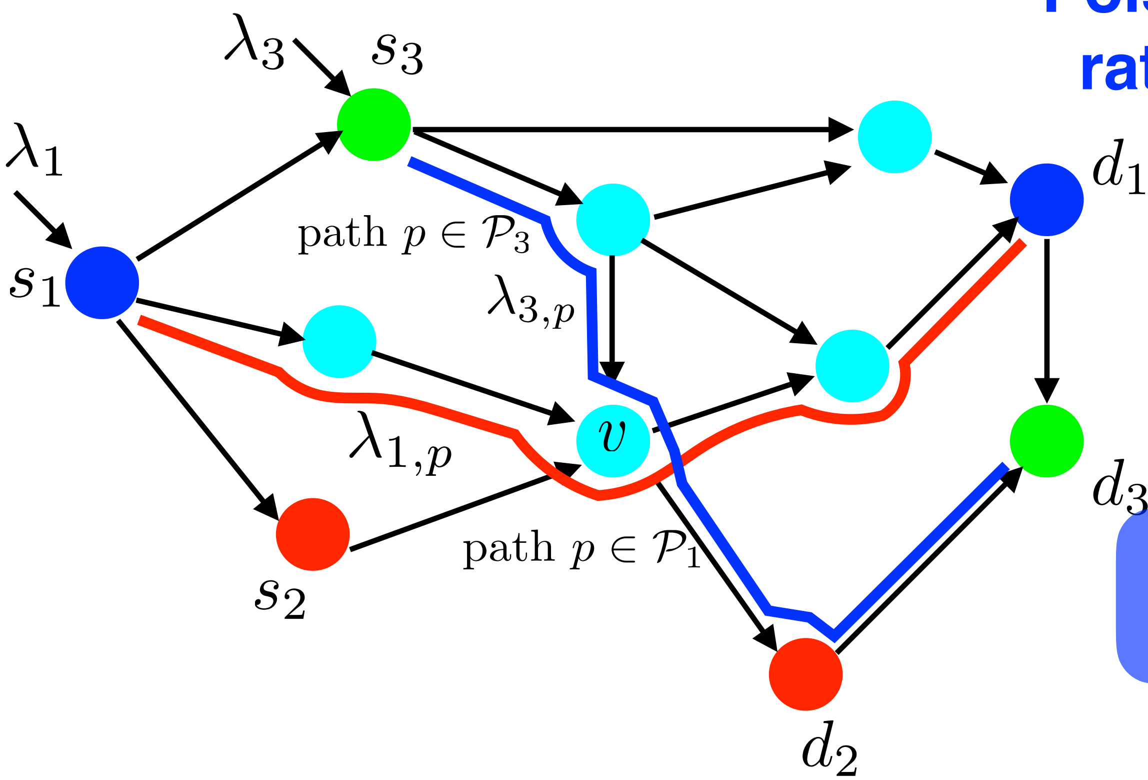

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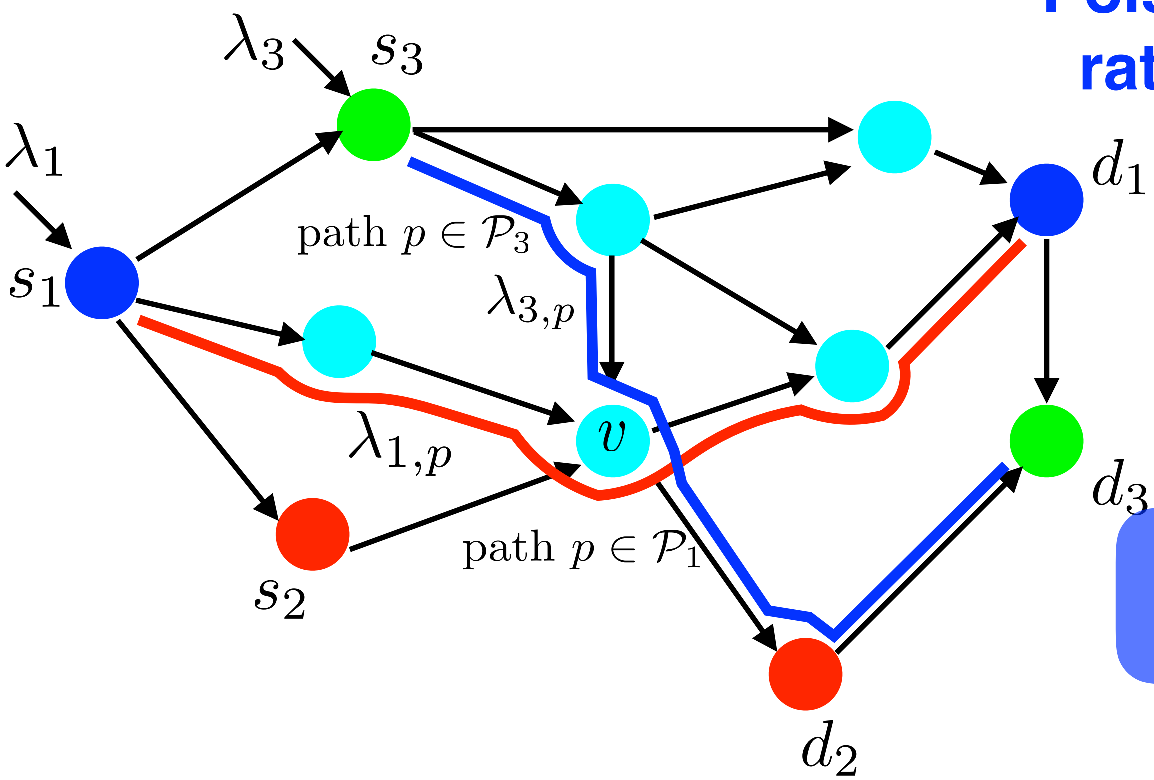
Algorithm



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A bit more than arrival rate

Easy to compute

Solve Convex Program : Total Power

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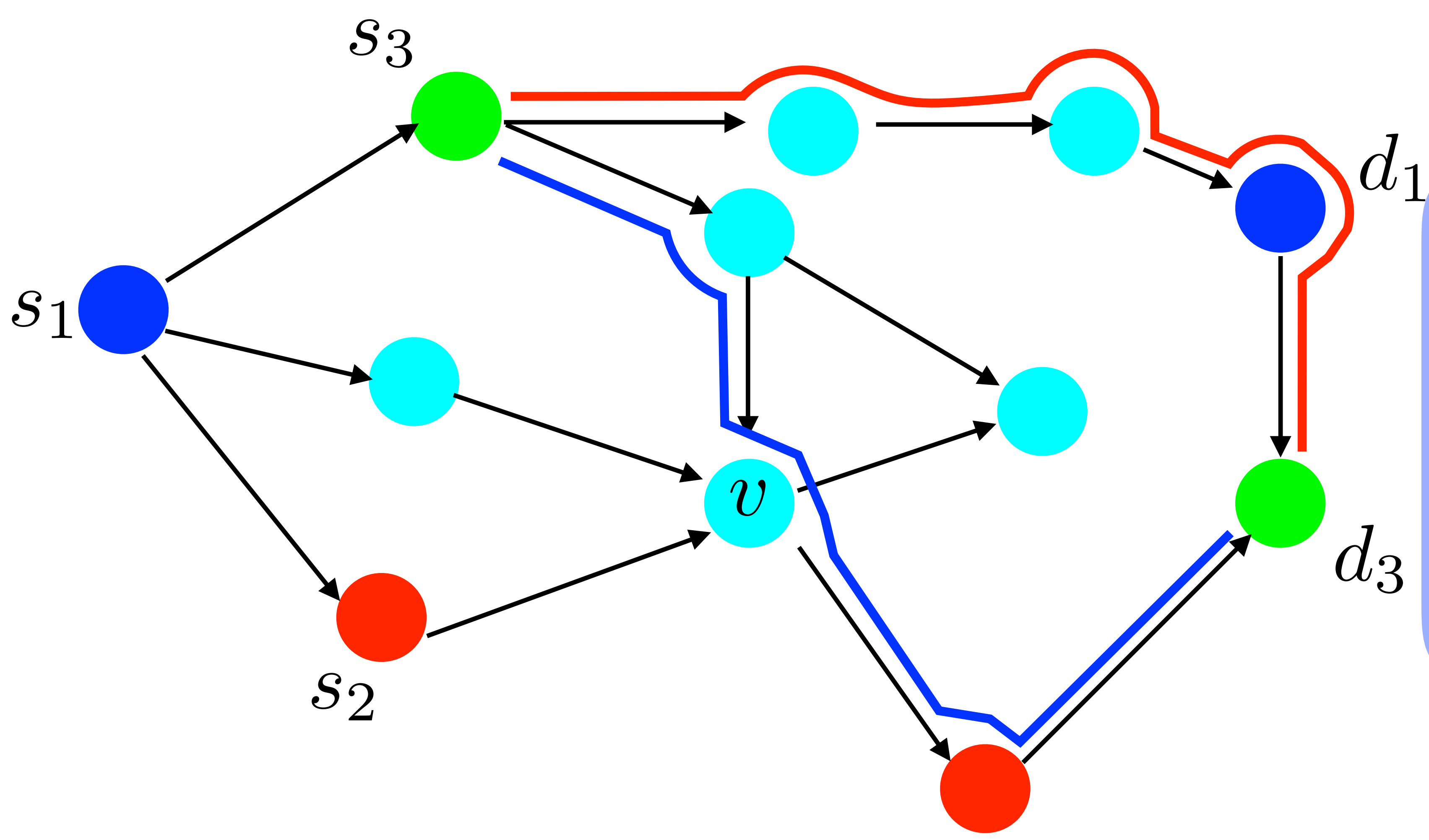
Interpreting the θ_{network}



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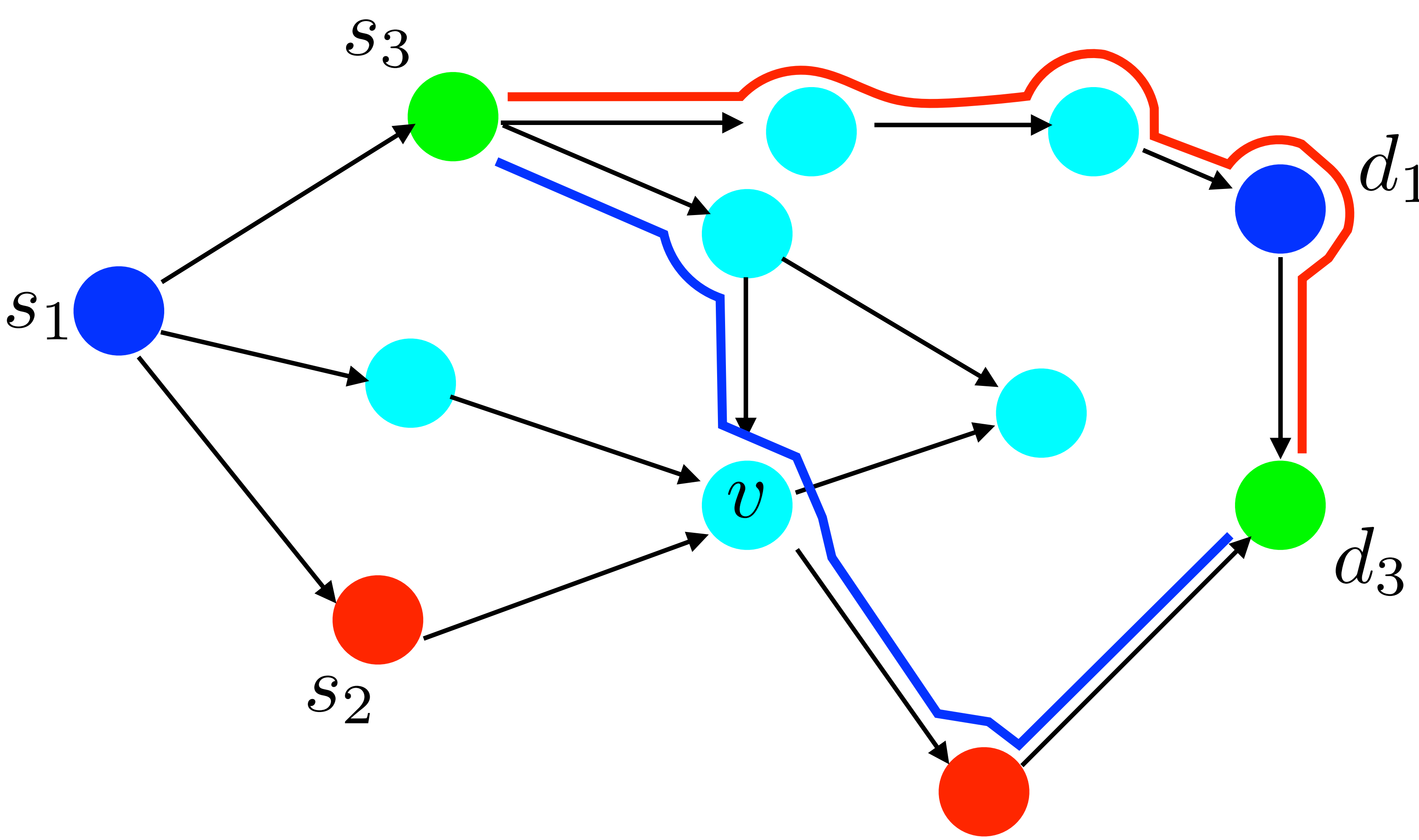

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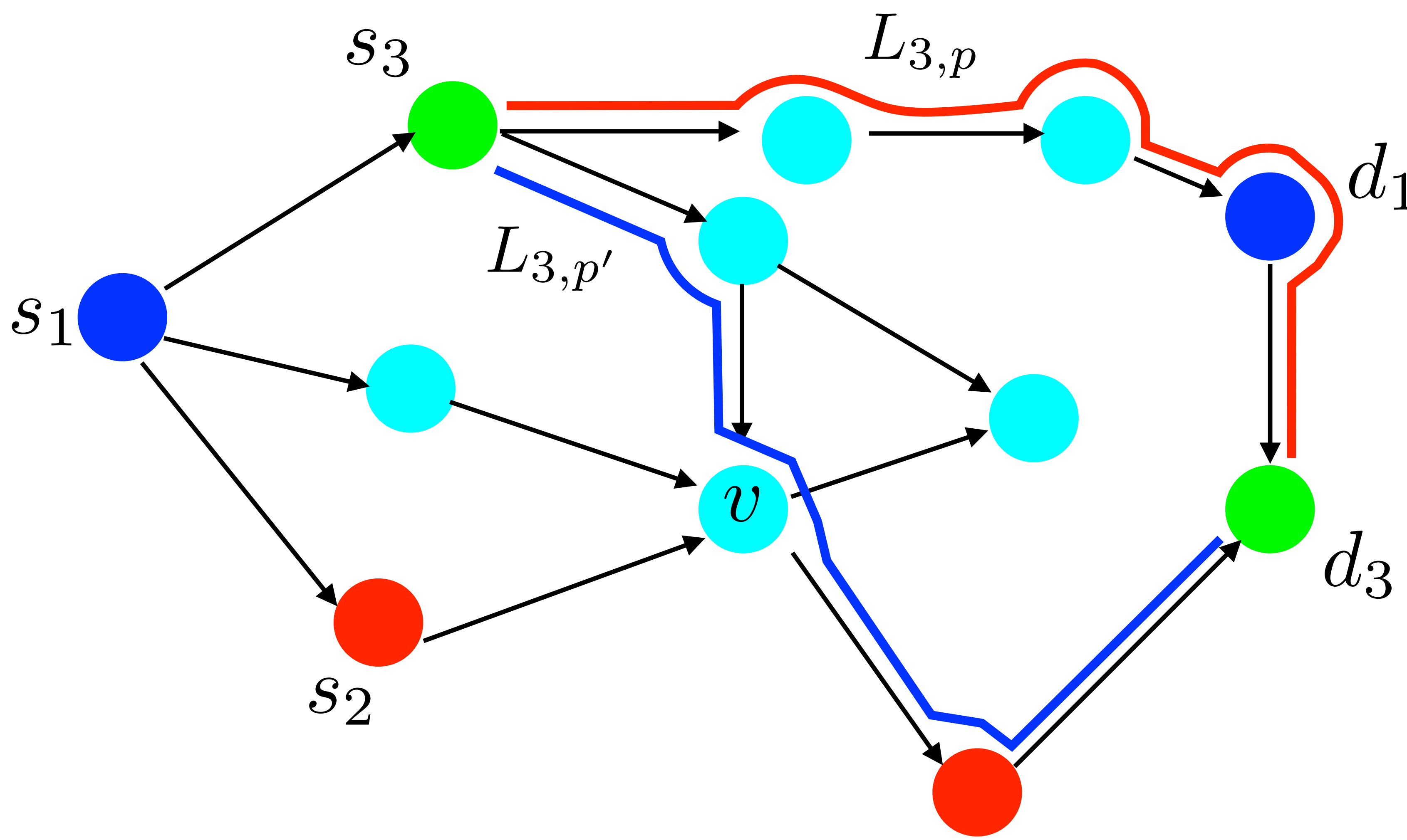

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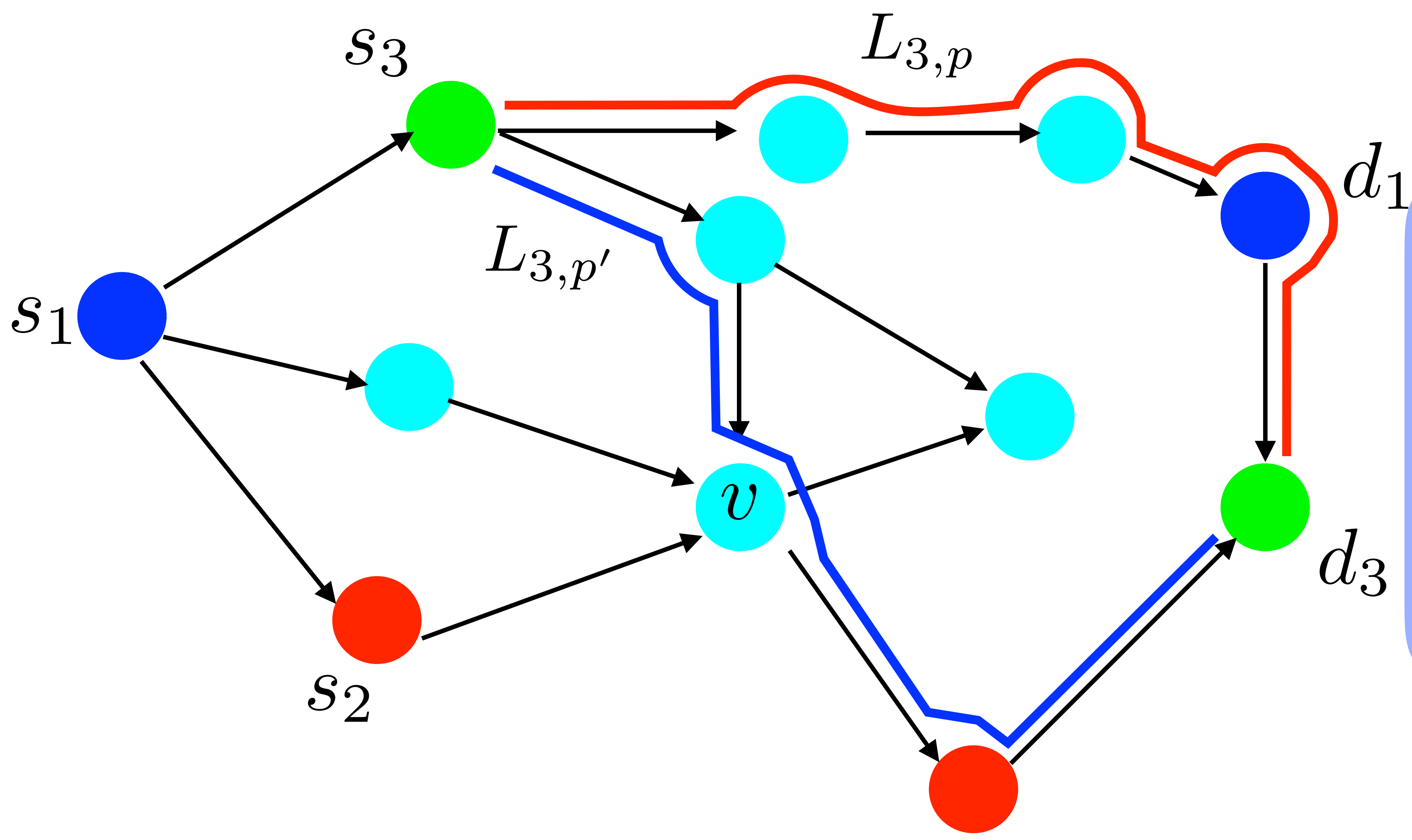

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i^{th} S-D pair

If $$\frac{\max_{p \in \mathcal{P}_i} L_p}{\min_{p \in \mathcal{P}_i} L_p} \leq c$$

Solve Convex Program : Total Power

$$\text{min.} \quad \sum_{v \in \mathcal{V}} \left(\sum_{i=1}^{\mathcal{D}} \sum_{p \in \mathcal{P}_i:\ v \in p} \lambda_{i,p} \right)^\alpha$$

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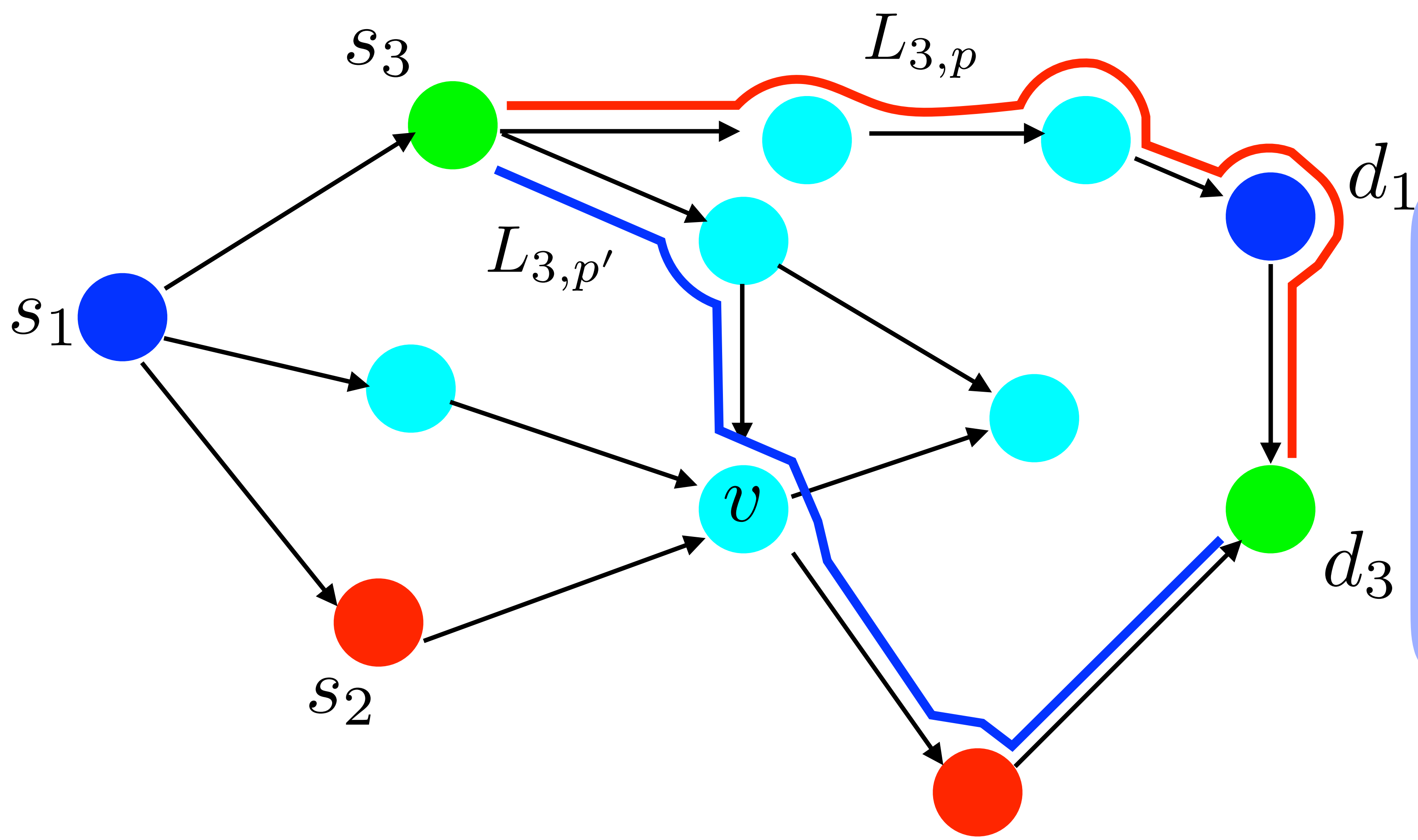

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i[th] S-D pair

If $\dfrac{\max_{p \in \mathcal{P}_i} L_p}{\min_{p \in \mathcal{P}_i} L_p} \leq c$

Then $\theta_i \leq c$

Solve Convex Program : Total Power

$$\text{min.} \quad \sum_{v \in \mathcal{V}} \left(\sum_{i=1}^{\mathcal{D}} \sum_{p \in \mathcal{P}_i:\ v \in p} \lambda_{i,p} \right)^{\alpha}$$

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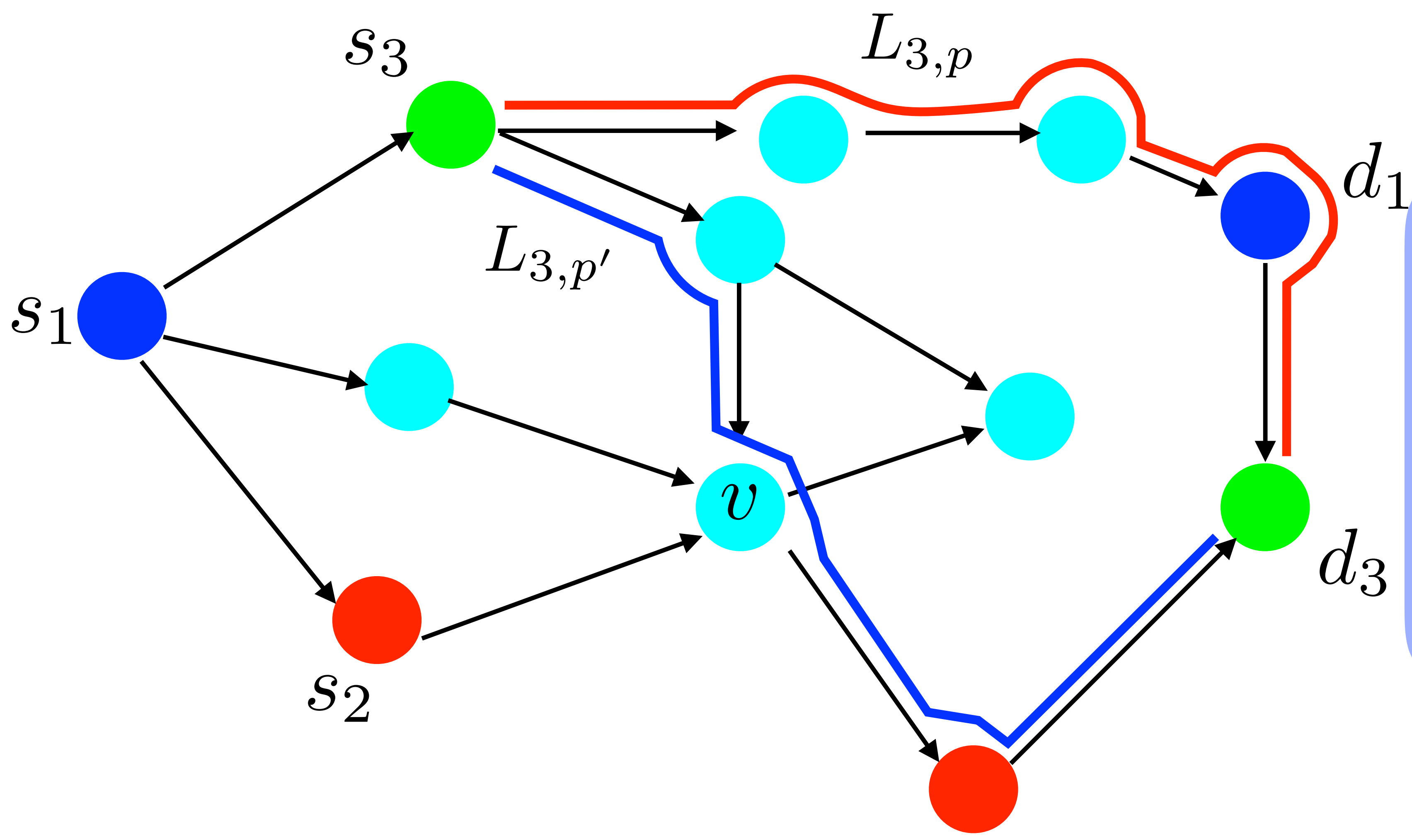

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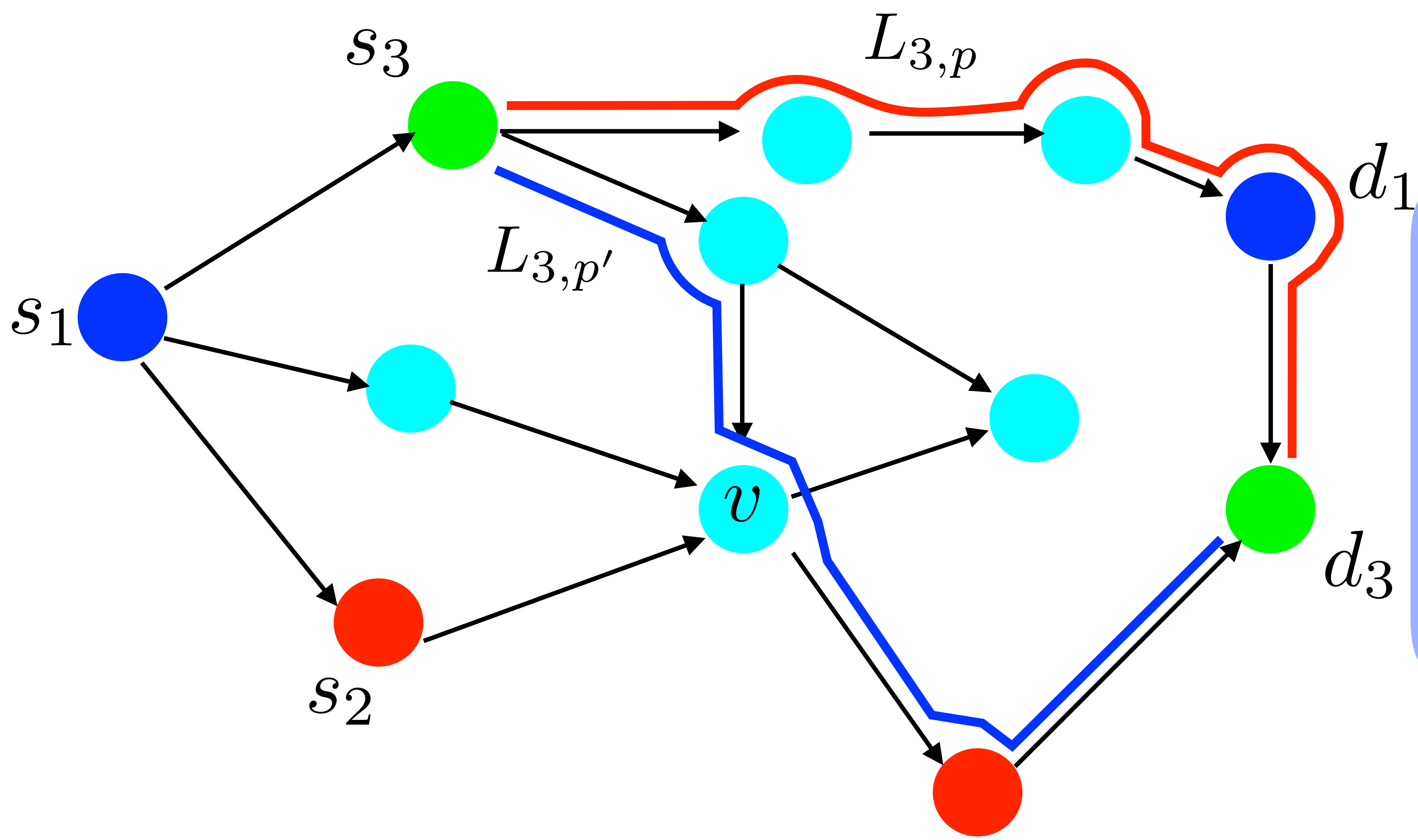
If $$\frac{\max_{p \in \mathcal{P}_i} L_p}{\min_{p \in \mathcal{P}_i} L_p} \leq c$$

Then $$\theta_i \leq c$$

Since $$\lambda_i = \sum_{\text{all paths from } s_i \to d_i} \lambda^\star_{i,p}$$

Interpreting the θ_{network}



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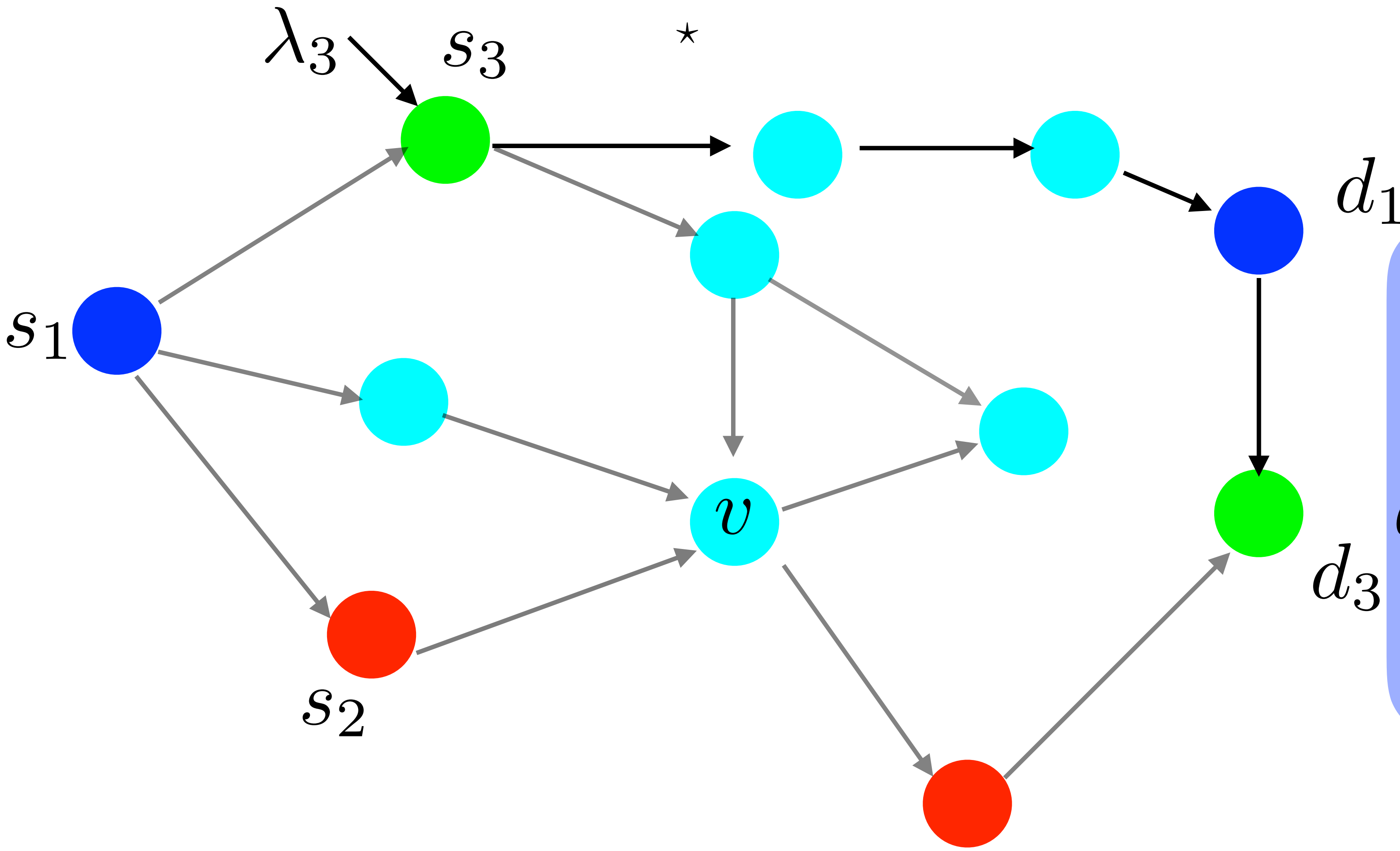

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$$\theta_{\text{network}} = \max_i \theta_i$$

Solve Convex Program : Total Power

$$\text{min.} \quad \sum_{v \in \mathcal{V}} \left(\sum_{i=1}^{\mathcal{D}} \sum_{p \in \mathcal{P}_i:\ v \in p} \lambda_{i,p} \right)^{\alpha}$$

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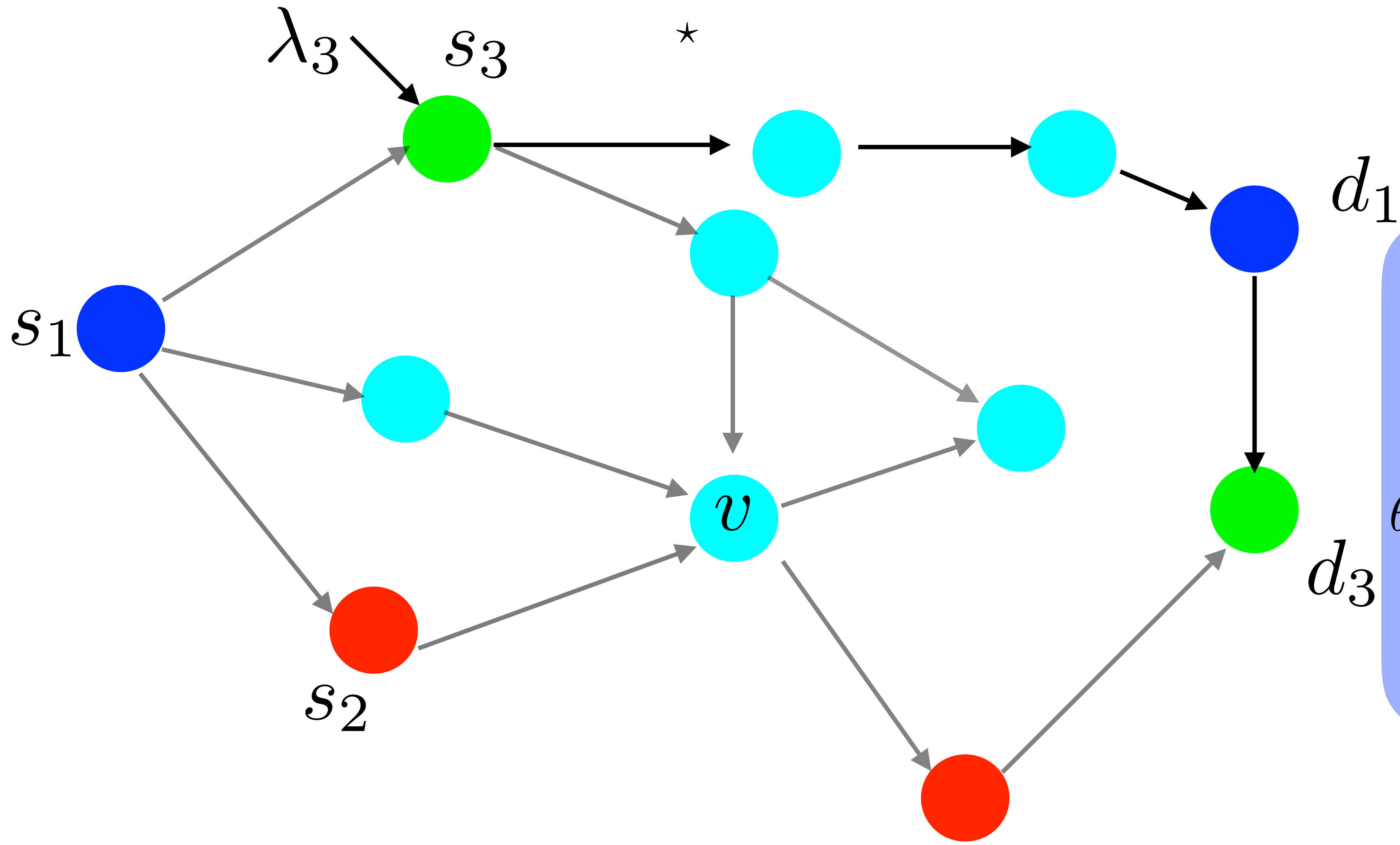
Interpreting the θ_{network}



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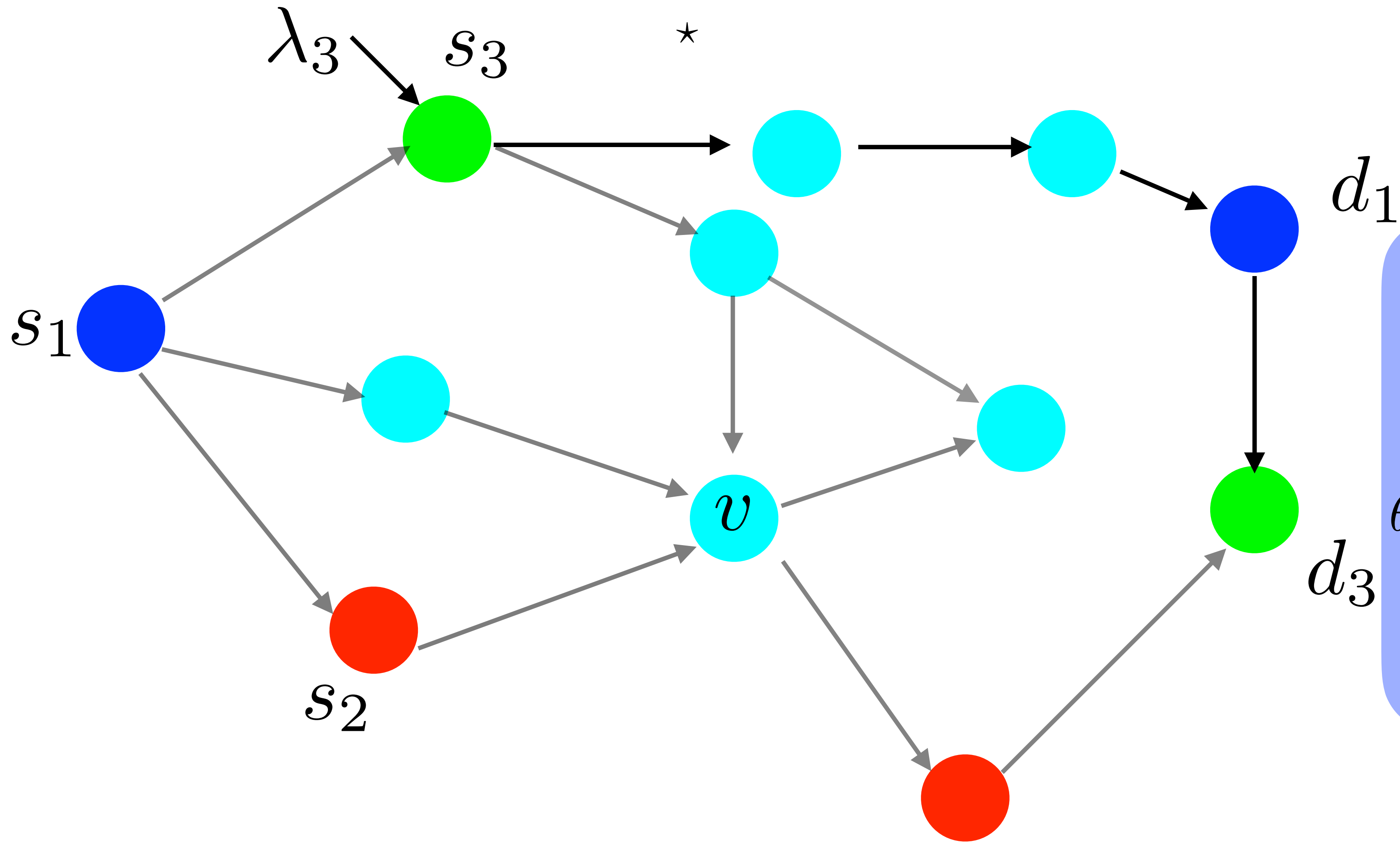

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$$\frac{L_{p'}}{\min_{p \in \mathcal{P}_i} L_p} = g(|V|)$$

For i^{th} s-d pair, paths p' is very long

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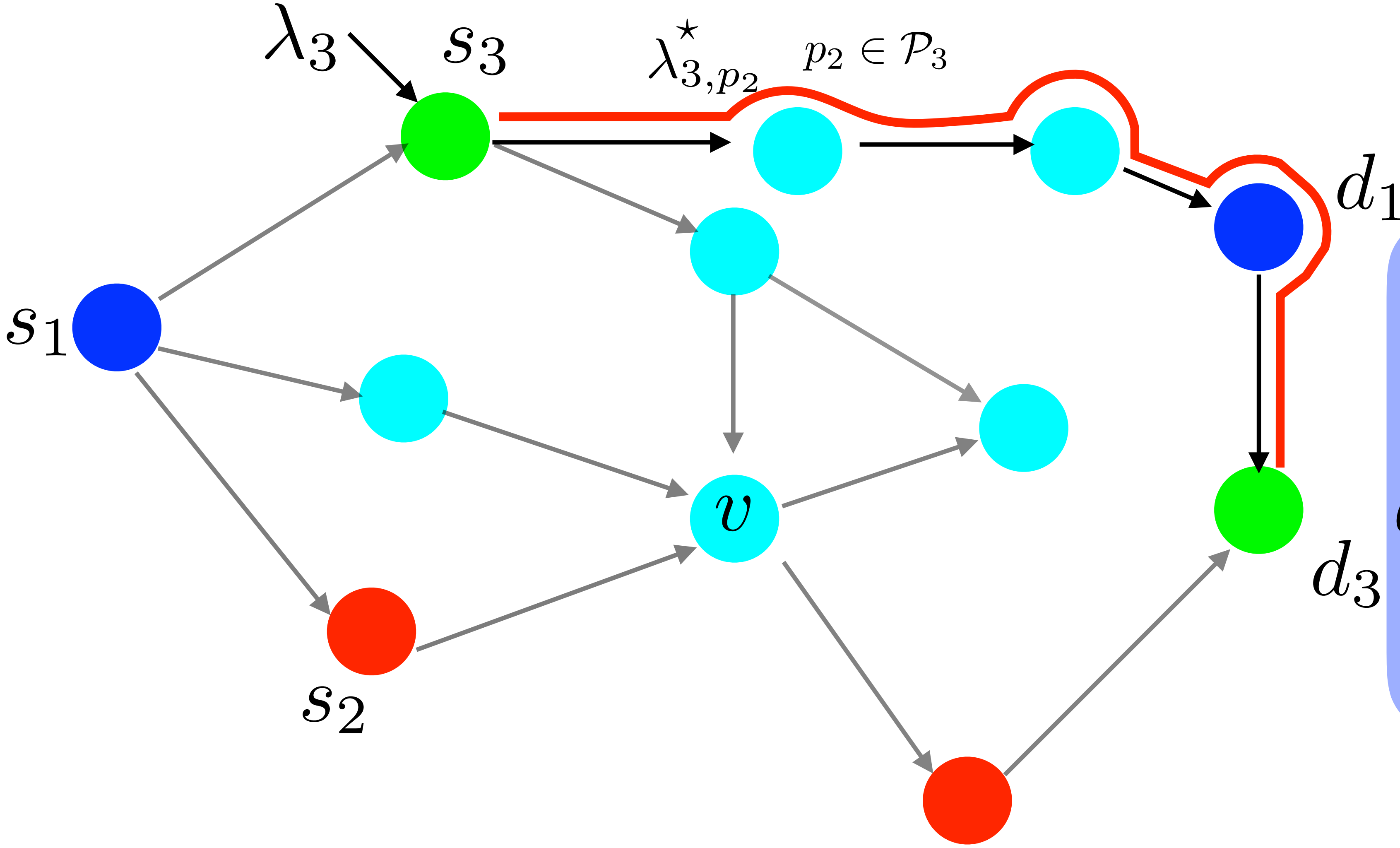

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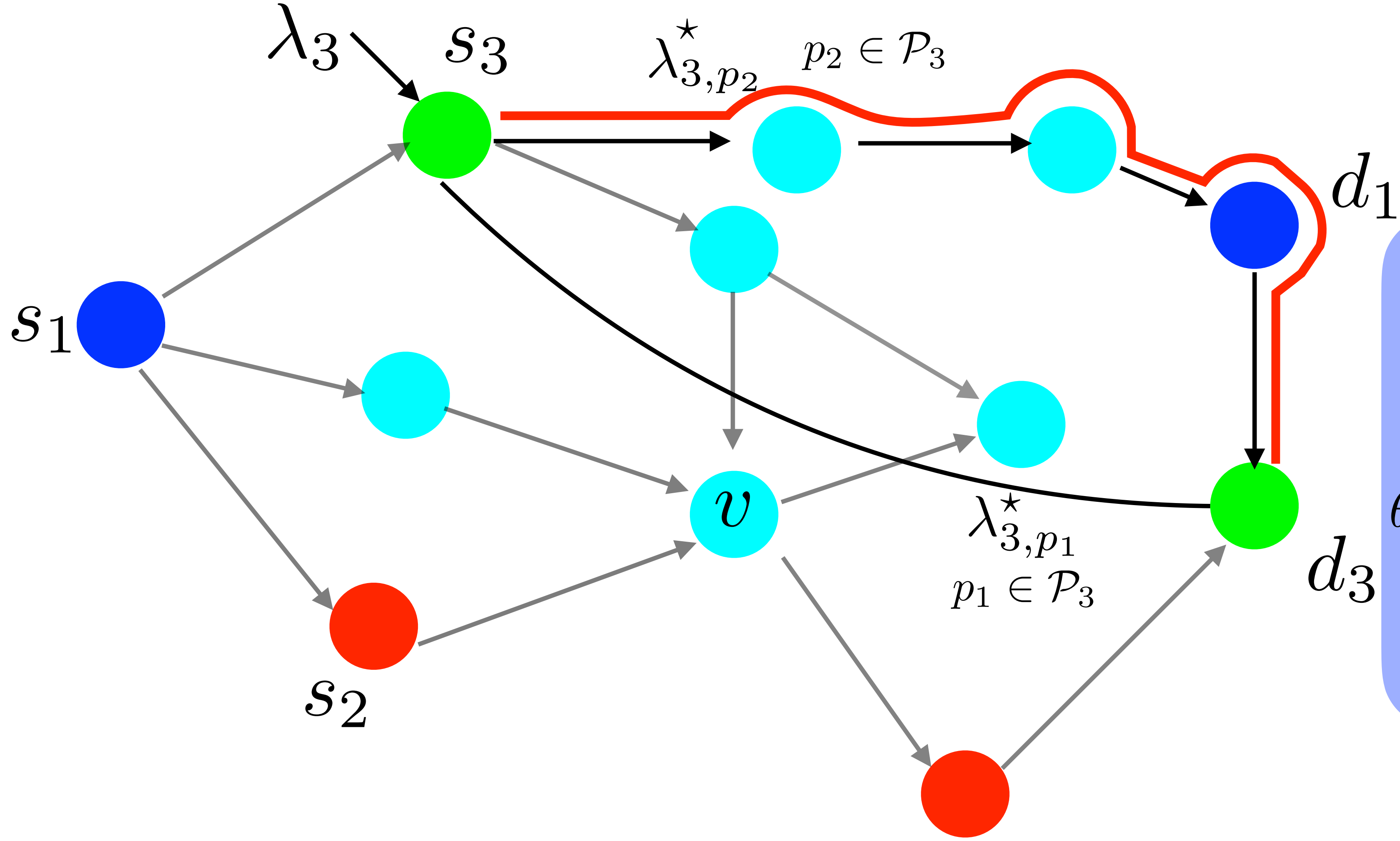

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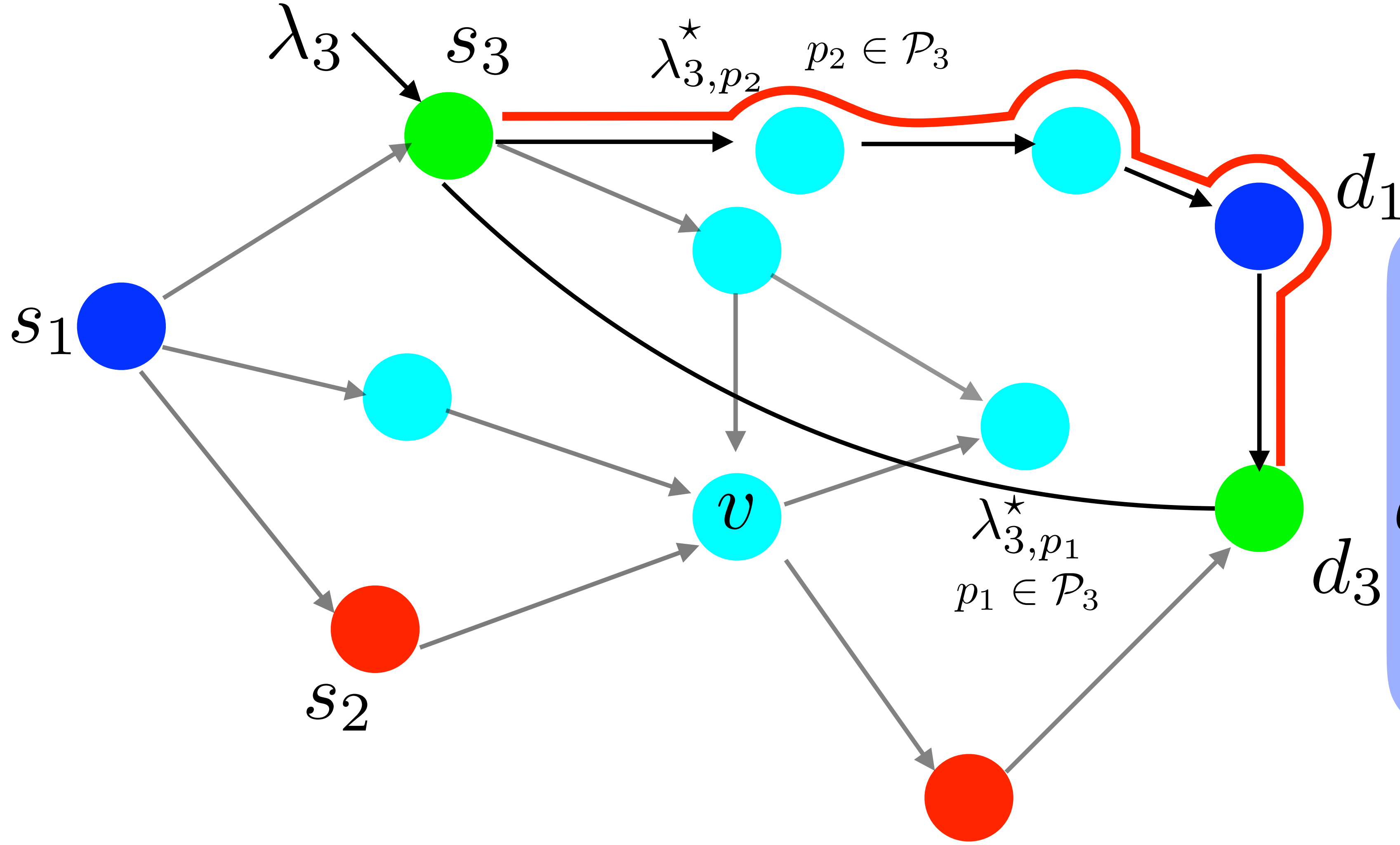

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$\lambda^\star_{i,p'}$ **For i^th s-d pair, paths *p'* is very long**

is likely to be small for longer path p'

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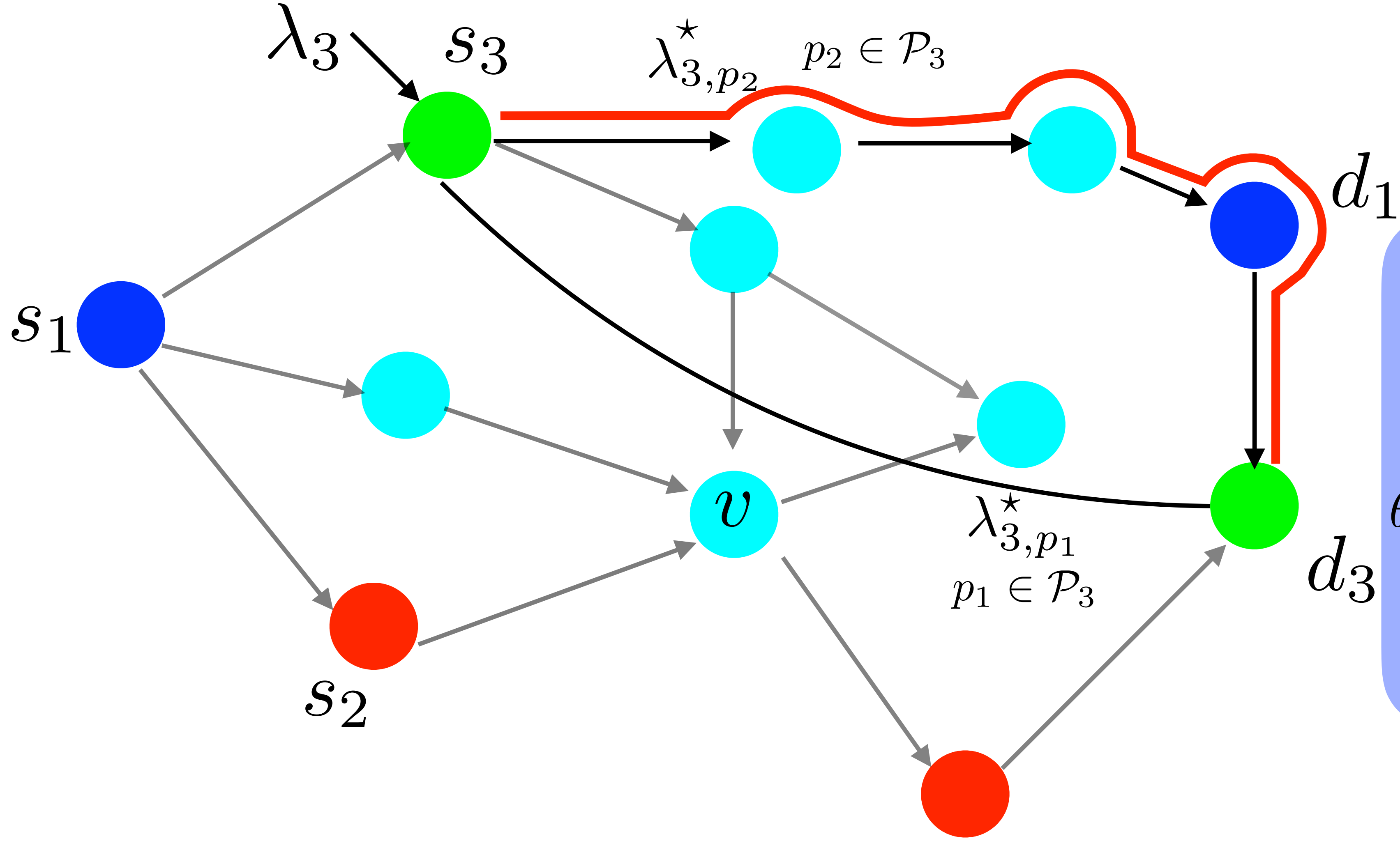

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Keeping θ_i small

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Worst Case Input

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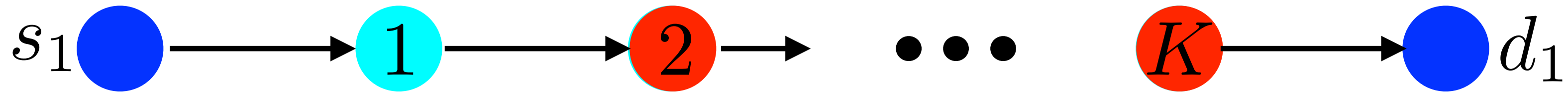
Line Network



Worst Case Input

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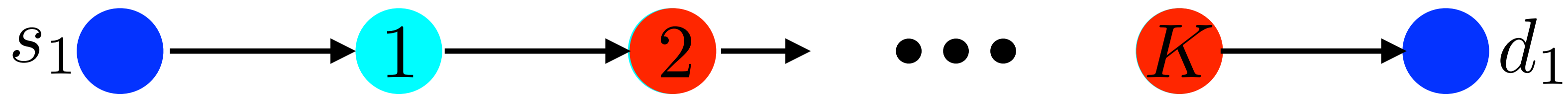
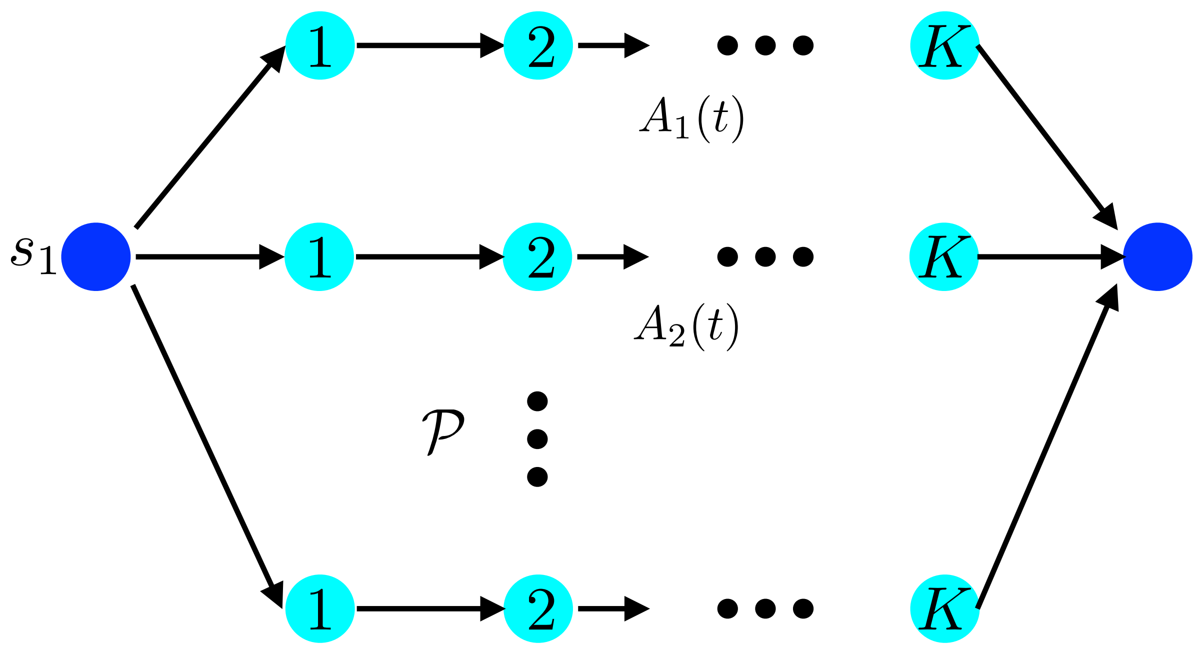


Series-Parallel



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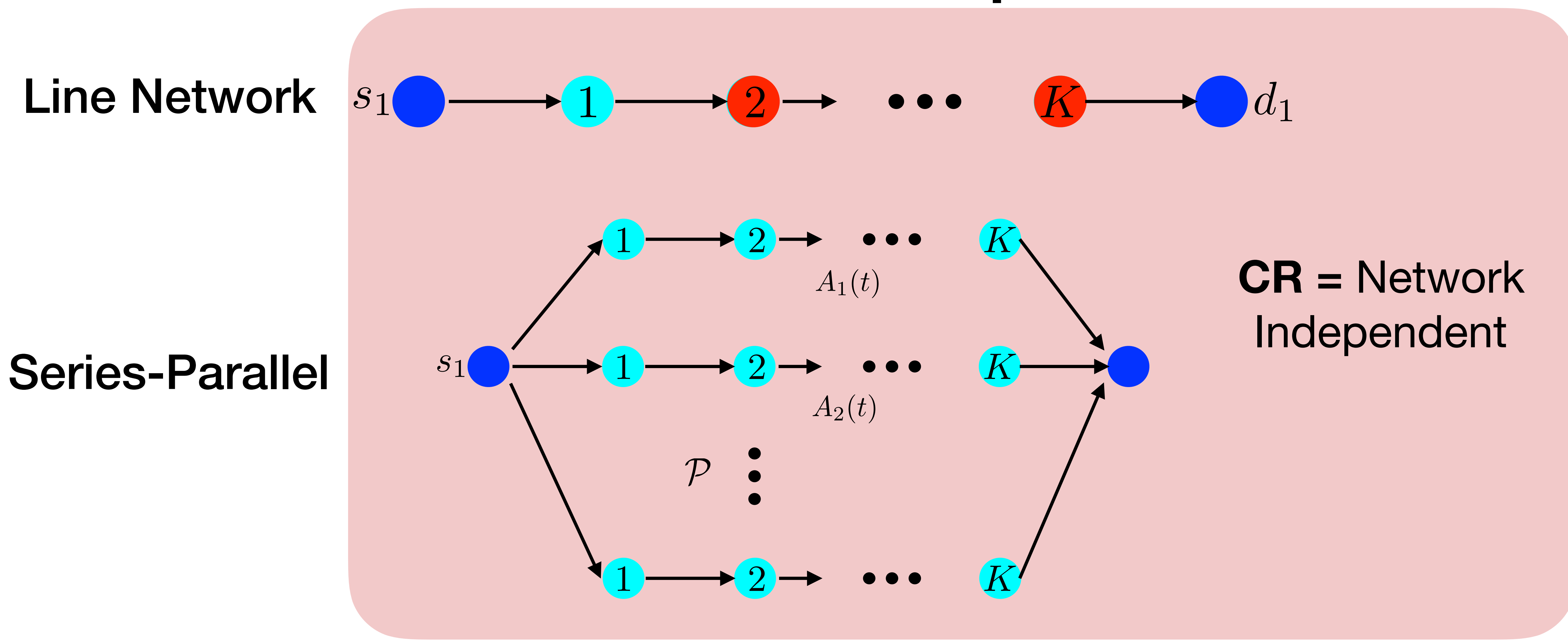

Worst Case Input

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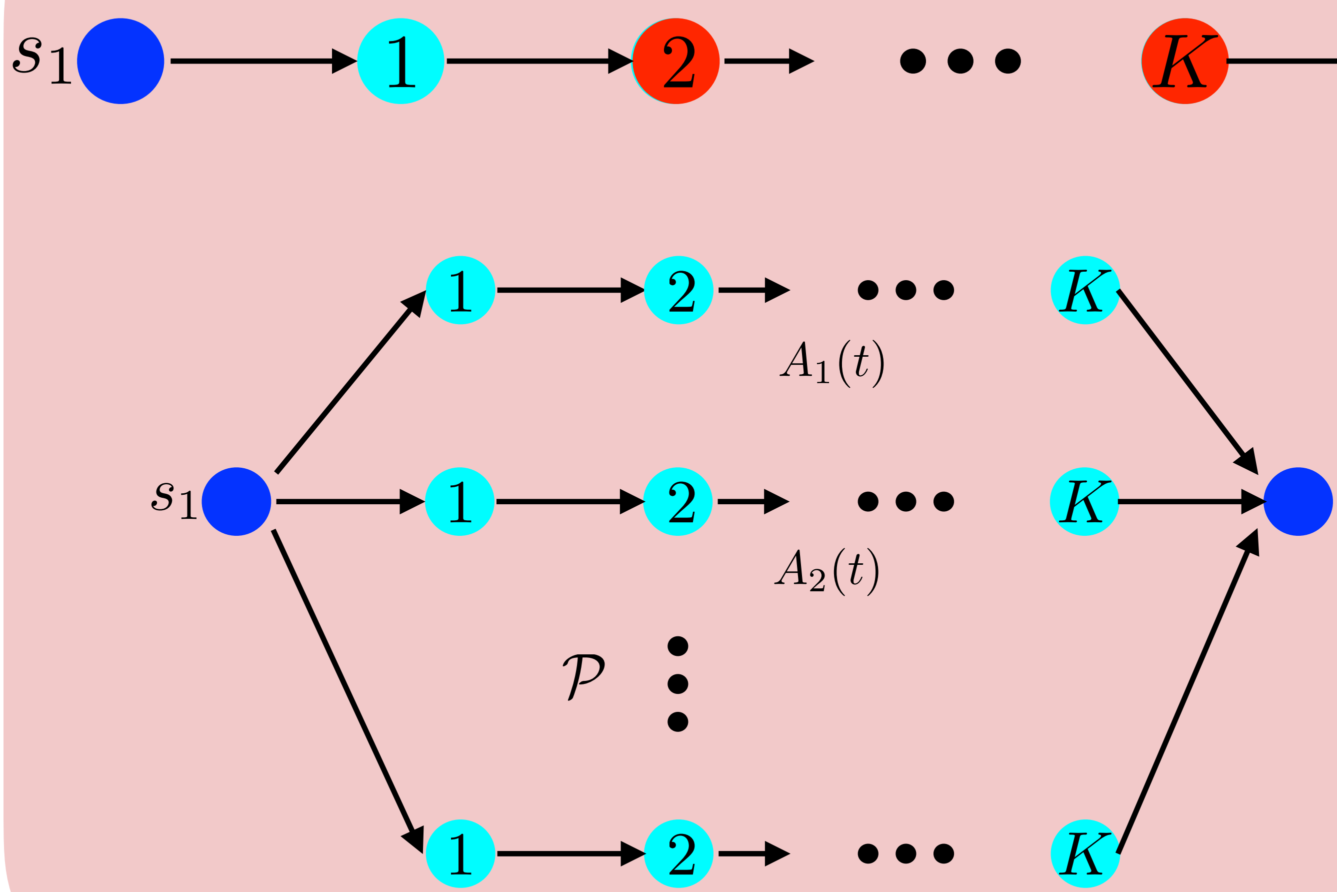
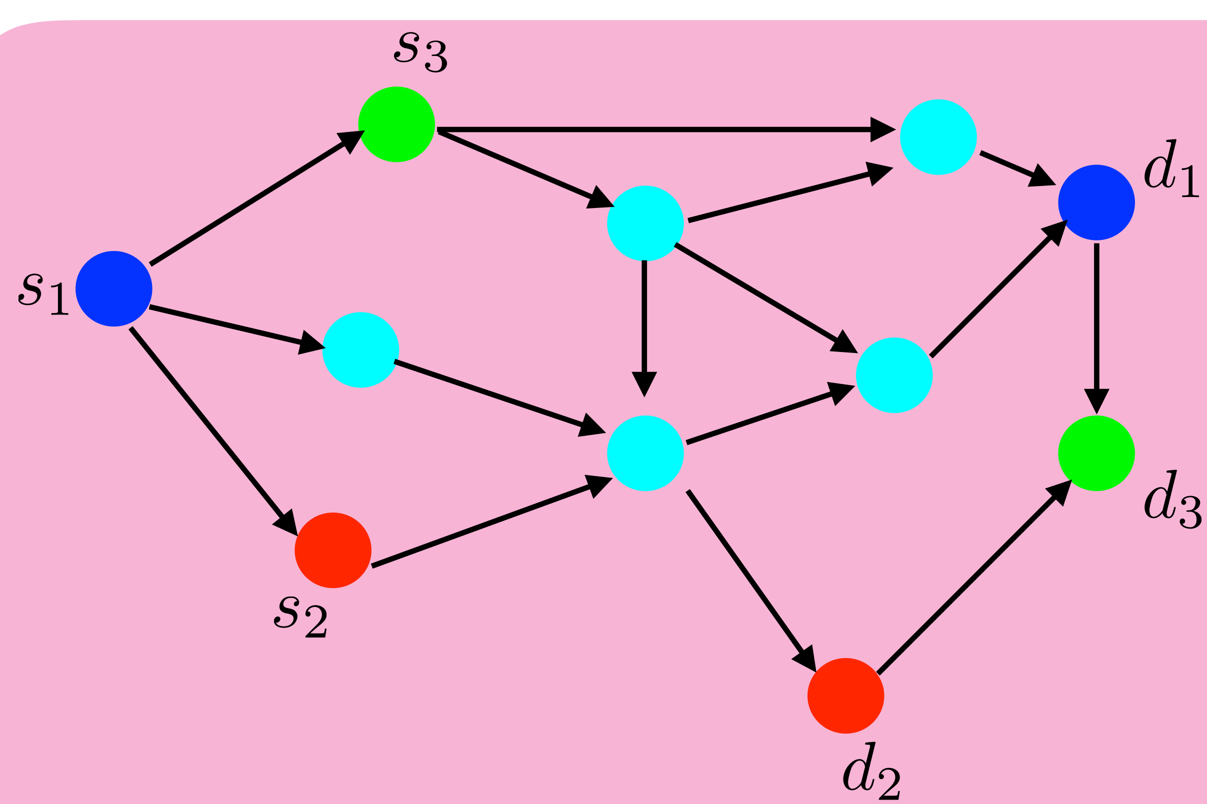
Series-Parallel



CR = Network Independent

SuperPosition of
Series-Parallel N/W



CR = O(deg$_{max}$)

Summary

Speed Scaling in Networks is difficult

Upper Bounds on Comp. Ratio

- **Stochastic Setting** — **For most 'nice' networks, CR is small**
- **Worst Case** — **For general networks, CR is on max congestion**

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Speed Scaling in Networks is difficult

Upper Bounds on Comp. Ratio

- **Stochastic Setting** **For most 'nice' networks, CR is small**
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Full-Talk

https://www.youtube.com/watch?v=BCXz5B96cEo&feature=youtu.be