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ABSTRACT

An objective of network neutrality is to design regulations for the Internet and ensure that it remains a public, open platform where innovations can thrive. While there is broad agreement that preserving the content quality of service falls under the purview of net neutrality, the role of differential pricing, especially the practice of zero-rating remains controversial. In this paper, we model zero-rating between Internet service providers (ISPs) and content providers (CPs) and show if zero-rating is permitted, the competitiveness in the market is reduced, where low-income CPs often lose utility and high-income CPs often gain utility.

Keywords  
Zero-Rating, Network Neutrality, Differential Pricing

1. INTRODUCTION

Net neutrality is the principle that Internet service providers (ISPs) treat all data on the Internet equally, and do not discriminate or charge differently by user, content, website, platform, type of equipment, or method of communication [1][2]. Although by some definitions, net neutrality does not include issues involving pricing, it is worthwhile to study how zero-rating impacts the market and how data is ultimately treated.

A commonly used practice of differential pricing is zero-rating: a service where ISPs do not charge customers for bandwidth consumed by specific applications and services, while customers pay the bandwidth price for other used services. Today, zero-rating is used in practice by particular cellular network providers [3]. Proponents of zero-rating argue that offering some services for free increases customer satisfaction and online services usage, as they can access more data at a given cost [4]. Critics argue that zero-rating allows financially prosperous CPs to pay for sponsored data, and adversely affects smaller CPs who cannot afford the same luxury [5][6], which jeopardizes the net neutrality rationale [7].

Arguments to date, such as the above, are qualitative, and there is no way to quantify the extent of the win or loss in a heterogeneous market of ISPs, CPs, and users based on enabling or disabling a zero-rated service.

In this paper, we formally model Internet settings where zero-rating is offered by ISPs to the CPs who deliver their content to users. We build our model to analyze how zero-rating impacts the market when ISPs, CPs, and consumers choose options that maximize their individual rewards. We perform 1) a macroscopic analysis, i.e. zero-rating impacts on the competitiveness of the market as a whole, 2) a microscopic analysis, i.e., zero-rating impacts on the behavior and decisions of individual ISPs and CPs. Our model differentiates CPs in terms of their value, i.e., how much revenue a CP makes per bandwidth unit used by their customers. While incumbents typically have higher values, startups have lower values, making less money per unit of bandwidth due to their smaller size and smaller market popularity. While our model and theoretical results are general for any kind of differential pricing, in this paper we tailor our analysis explicitly to the zero-rating context, as it is the only prevalent real-world implementation of the late. The new knowledge can guide regulators to design better policies to address net neutrality issues from an interconnection context.

Our contributions and conclusions are as follows.

- We consider both ISPs’ and CPs’ zero-rating decisions as a bargaining problem, analyze their strategic behavior, and introduce the concept of zero-rating equilibria.
- We identify the zero-rating pressure phenomenon where CPs only zero-rate when their competitors do so, and find how it impacts CPs’ decisions and their utilities.
- We analyze the impact of zero-rating on the market of CPs both globally by analyzing the Herfindahl index [8], and individually by analyzing CPs’ utilities.
- We numerically explore the parameter space of our model and demonstrate the impact of zero-rating on market shares and profitability of the CPs under varying market conditions.

The rest of the paper is organized as follows. Section 2 builds the choice model which takes ISPs and CPs as complementary services and characterizes their market shares (Equation 2). Section 3 builds the utility model (Equation 3) under various zero-rating and market structures. Section 4 theoretically analyzes the Herfindahl index and utilities under zero-rating equilibria, and Section 5 numerically measures the Herfindahl index and CPs’ utilities under the equilibrium in a duopolistic market of ISPs/CPs. Section 6 presents the related work, and finally the paper is concluded in Section 7.

2. MODEL

We consider a setting with 3 types of players: user, ISP, and CP. A user selects one of |M| ISPs as their bandwidth provider and can choose from one or more of |N| CPs as their content provider. We compute each ISP’s and CP’s income based on the bandwidth unit of data consumed from it.

In some cases, there exist users who may not choose any ISPs or any CPs. Furthermore, some users may utilize multiple CPs at a time. In other words, each user may choose a combination of the existing CPs with a certain probability. To facilitate the analysis,
rather than having users who select no ISP or no CP (or neither), we assume the existence of a dummy CP and a dummy ISP. Users who would select no ISP (CP) can be mapped to a setting where they select the dummy ISP (CP) at no cost. Note that the dummy ISP carries no traffic and the dummy CP has no content to offer.

A user, therefore, has a choice of \(|\mathcal{M}| = |\mathcal{M}| + 1\) ISPs, including the dummy ISP, and chooses one of \(|\mathcal{N}| = 2^{|\mathcal{N}|}\) possible subsets of CPs for their content, including the null set (the dummy CP). We refer to the subset of CPs chosen by the user as auxiliary CPs and denote the set of ISPs and CPs (including dummies and auxiliaries) by \(\mathcal{M}\) and \(\mathcal{N}\), respectively. Thus, a user always picks an ISP from \(\mathcal{M}\) for her Internet access and a CP from \(\mathcal{N}\) for the content.

### 2.1 Zero-Rating

In a paradigm where zero-rating is permitted, an ISP and a CP may agree that instead of the user, the CP will cover the bandwidth cost of the content viewed by the user, potentially at a price lower than what the user would pay. A user’s choice of ISPs and CPs could depend on whether zero-rating is permitted and offered by an ISP-CP pair as a service.

We assume customers are mapped to an ISP-CP pair \((i,j)\) according to a pre-determined distribution. If ISP \(j\) and CP \(i\) zero-rate, the probability of customers being assigned to them changes. We denote the zero-rating relationship between CP \(i\) \(\in\mathcal{N}\) and ISP \(j\) \(\in\mathcal{M}\) by \(\theta_{ij} \in \{0,1\}\), where \(\theta_{ij} = 1\) indicates zero-rating between \(i\) and \(j\) is established, otherwise \(\theta_{ij} = 0\).

Even though zero-rating does not apply to the dummy ISP \(i = 0\) or prep rate \(j = 0\), we always assume \(\theta_{0j} = \theta_{i0} = 0\); \(\forall i \in \mathcal{N}, j \in \mathcal{M}\). On the other hand, each auxiliary CP is assumed to zero-rate with any given ISP \(j\) if and only if all the CPs comprising it zero-rate with the ISP.

### 2.2 Complementary Choices Model \((\mathcal{N}, \mathcal{M})\)

We denote the baseline market share of ISP \(j\) by \(\psi_{j} \in [0,1]\), which captures the intrinsic characteristics such as price and brand name, and models the market share of ISP \(j\) when none of ISPs in the system zero-rate with the CPs. We also denote the baseline market share of CP \(i\) by \(\phi_{i} \in [0,1]\), i.e., the market share of CP \(i\) when none of CPs zero-rate with the ISPs, and \(\sum_{i\in\mathcal{N}} \phi_{i} = 1\).

We define \(\psi = (\psi_{1}, \cdots, \psi_{|\mathcal{M}|})^{T}\) and \(\phi = (\phi_{1}, \cdots, \phi_{|\mathcal{N}|})^{T}\). The zero-rating matrix of the whole system is defined as \(\Theta = \{\theta_{ij} : i \in \mathcal{N}, j \in \mathcal{M}\}\). Furthermore, we define \(\alpha\) as the fraction of elastic users in the market who choose among the CPs and ISPs with zero-rating relations, and if no such providers exist, these users would be distributed among all the providers. The rest of the users, denoted as sticky users, are distributed among CPs and ISPs merely based on their baseline market shares and independent of the zero-rating relations.

In practice, users may choose services from constrained sets of CPs and ISPs. In general, we denote a set of choice pairs by \(\mathcal{L}\). This set of available choices is impacted by \(\Theta\).

**Assumption 1.** The set \(\mathcal{L}\) for sticky users is the entire choice set \(\mathcal{N} \times \mathcal{M}\). The set \(\mathcal{L}\) for elastic users is any pair of CP \(i\) and ISP \(j\) who zero-rate with one another, and if there is no such pair, it would be the entire choice set \(\mathcal{N} \times \mathcal{M}\).

**Assumption 2.** Given a nonempty set \(\mathcal{L}\) of available choices, a user chooses a choice pair \((i,j)\) \(\in\mathcal{L}\) with probability

\[
\mathbb{P}_{\mathcal{L}}\{i,j\} = \frac{\phi_{i}\psi_{j}}{\sum_{(n,m)\in\mathcal{L}} \phi_{n}\psi_{m}}
\]

(1)

The zero-rating matrix of the whole system is defined as \(\Theta = \{\theta_{ij} : i \in \mathcal{N}, j \in \mathcal{M}\}\). We denote the zero-rating relationship between CP \(i\) and ISP \(j\) according to a pre-determined distribution. If ISP \(j\) and CP \(i\) zero-rate, we assume the existence of a dummy CP and a dummy ISP. Users who would select no ISP (CP) can be mapped to a setting where they select the dummy ISP (CP) at no cost. Note that the dummy ISP carries no traffic and the dummy CP has no content to offer.

A user, therefore, has a choice of \(|\mathcal{M}| = |\mathcal{M}| + 1\) ISPs, including the dummy ISP, and chooses one of \(|\mathcal{N}| = 2^{|\mathcal{N}|}\) possible subsets of CPs for their content, including the null set (the dummy CP). We refer to the subset of CPs chosen by the user as auxiliary CPs and denote the set of ISPs and CPs (including dummies and auxiliaries) by \(\mathcal{M}\) and \(\mathcal{N}\), respectively. Thus, a user always picks an ISP from \(\mathcal{M}\) for her Internet access and a CP from \(\mathcal{N}\) for the content.

### 2.3 User Model

Our complementary choices model \((\mathcal{N}, \mathcal{M})\) can be specified by a triple of vectors \((\phi, \psi, \Theta)\) and the scalar \(\alpha\). We denote the total market size by \(X\). We introduce the number of users of \((i,j)\) denoted by \(X_{ij}\) based on Assumptions 1 and 2. \(X_{ij}\) is a function of \(\Theta\) where \(X_{ij}(\Theta) = \rho_{ij}(\Theta)X\), where \(\rho_{ij}\) is the closed form market share of the pair \((i,j)\) and we have:

\[
\rho_{ij}(\Theta) = \left[ \alpha \times \sum_{i,j} \frac{\phi_{i}\psi_{j}\theta_{ij}}{\sum_{i,j} \phi_{i}\psi_{j}\theta_{ij}} + (1 - \alpha) \times \phi_{i}\psi_{j} \right] 1_{\{i,j\}}
\]

(2)

Equation 2 derives the number of users \(X_{ij}\) for any pair \((i,j)\) of complementary providers under the zero-rating profile \(\Theta\). If CP \(i\) and ISP \(j\) do not zero-rate \((\theta_{ij} = 0)\), the pair \((i,j)\) of providers keep the proportion \(1 - \alpha\) of their sticky users, who are distributed in the system based on baseline market shares. If CP \(i\) and ISP \(j\) zero-rates with ISPs \(\theta_{ij} = 1\), not only they will keep the proportion \(1 - \alpha\) of their sticky users, but also the elastic users, which comprise a fraction of users would choose this pair with probability proportional to their baseline market shares. If none of the CPs and ISPs offer zero-rating, all users are again distributed among the providers based on their baseline market shares.

**Lemma 1.** Let \(N' \subseteq N\) and \(M' \subseteq M\). For any \(n \in N\) and \(m \in M\), let \(\tilde{N} \triangleq \tilde{N}\backslash\{n\} \cup \{n'\}\) and \(\tilde{M} \triangleq \tilde{M}\backslash\{m\} \cup \{m'\}\) denote the new sets of providers where the subsets \(N'\) and \(M'\) are replaced by the providers \(n\) and \(m\), respectively. Let \(X_{ij}\) denote the number of users of \((i,j)\) under \((\tilde{N}, \tilde{M})\). If \(\theta_{ij} = \alpha\), \(\forall i \in N', j \in M'\), then

\[
\tilde{X}_{nm} = \sum_{i \in N'} \sum_{j \in M'} X_{ij}, \quad \tilde{X}_{ij} = X_{ij}, \quad \forall i \neq n, j \neq m;
\]

\[
\tilde{X}_{nm} = \sum_{i \in N'} X_{ij}, \forall j \neq m; \quad \tilde{X}_{im} = \sum_{j \in M'} X_{ij}, \forall i \neq n.
\]

Lemma 1 states that if there exists multiple CPs (or ISPs) that use the same zero-rating profile, then they could be conceptually merged as a single CP (or ISP) without affecting the market shares of other providers.

When the users choose one ISP and multiple CPs at the same time, they contribute to the revenues of the ISP and all CPs they choose. Let’s assume that the set of auxiliary CPs who are comprised from the actual CP \(i\) is shown by \(AU(i)\). Therefore, the effective

<table>
<thead>
<tr>
<th>parameter</th>
<th>description</th>
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<tbody>
<tr>
<td>((x,y))</td>
<td>a pair of CP (x) and ISP (y)</td>
</tr>
<tr>
<td>(N, M)</td>
<td>set of all CPs, ISPs (including auxiliary)</td>
</tr>
<tr>
<td>(\theta_{ij})</td>
<td>zero-rating relation between ((i,j))</td>
</tr>
<tr>
<td>(X_{ij})</td>
<td>#users of ((i,j)); (i \in N, j \in M)</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>user elasticity</td>
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<tr>
<td>(c)</td>
<td>bandwidth usage coefficient for non-zero-rated data</td>
</tr>
<tr>
<td>(p_{i})</td>
<td>per bandwidth revenue of CP (i); (i \in M)</td>
</tr>
<tr>
<td>(q_{j})</td>
<td>per bandwidth price of ISP (j); (j \in M)</td>
</tr>
<tr>
<td>(\phi_{i})</td>
<td>baseline market share of CP (i); (i \in N)</td>
</tr>
<tr>
<td>(\psi_{j})</td>
<td>baseline market share of ISP (j); (j \in M)</td>
</tr>
<tr>
<td>(\delta_{ij})</td>
<td>price discount of ISP (j); (j \in M)</td>
</tr>
<tr>
<td>(U_{1}, R_{j})</td>
<td>utility of CP (r), ISP (j); (i \in N, j \in M)</td>
</tr>
</tbody>
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Table 1: Summary description of parameters.
number of users who utilize the pair of and actual CP $i$ and ISP $j$

\[ X_{ij} = \sum_{i \in A} X_{ij} \quad (3) \]

**Corollary 1.** When all the providers $N \times M - \{(i, j)\}$ have fixed strategies, zero-rating $(i, j)$, i.e., changing $\theta_{ij}$ from 0 to 1, always helps CP $i$ to attract more customers.

Based on Equation \[2\] in case CP $i$ zero-rates with ISP $j$, since $\theta_{ij} = 1$, as the first term of the equation is a positive value, it causes $X_{ij}$ to increase, which then increases $X_{ij}$ based on Equation \[3\]

### 3. Utility and Zero-Rating Equilibrium

We assume each ISP, by choosing its pricing structure, decides along with each CP whether or not to adopt zero-rating for their customers. While the data prices and values are assumed to be exogenous, each ISP $j$ has the option of charging CPs a different data price $\delta p_j$ if they zero-rate, where $0 < \delta \leq 1$ denotes the data price discount ISP $j$ offers to the CPs. If $\delta < 1$, CPs can purchase ISP $j$’s bandwidth as a zero-rated service with a lower price than the users can directly purchase it. Each ISP $j$ should strategically choose this price discount, as for some $\delta_j$, its total revenue could increase even though its bandwidth unit income decreases, since it could attract a higher number of customers. However, a very low value of $\delta_j$ could harm ISP $j$’s total revenue. We define $\delta \triangleq (\delta_1, \ldots, \delta_M)^T$ to denote the entire ISP discount profile of the market.

In this section, we analyze ISPs’ and CPs’ zero-rating decisions. Note that although the auxiliary providers do not make independent zero-rating decisions, we account for their users in our evaluations since they contribute to the users of actual providers (Definition \[3\]), and therefore to their utilities. However, the users of dummy providers do not generate any utilities. We first introduce the following assumption to compute the utility model of the actual providers in the market.

**Assumption 3.** The revenue of ISP $j \in M$ from the market of CPs is equal to the summation of revenues each CP $i$’s user brings to $j$ for all $i \in N$. Likewise, the utility\(^\dagger\) of each actual CP $i \in N$ is equal to the summation of utilities each ISP $j$’s user brings to $i$ for all $j \in M$.

**Assumption 4.** When zero-rating is provided for a pair of CP $i$ and ISP $j$, i.e., $\theta_{ij} = 1$, since users would not pay the bandwidth price, their average bandwidth usage may increase by a factor of $1/c$, where $0 < c \leq 1$.

When the bandwidth usage increases in case of zero-rating, the utilities of CPs and the revenues of ISPs who zero-rate will also increase since they are a function of per-bandwidth unit prices. To model this, instead of assuming the utilities and revenues of providers who zero-rate increase by a factor $1/c \geq 1$, for simplicity we assume if the providers cancel their zero-rating, their utilities and revenues decreases by a factor of $0 \leq c \leq 1$, which we call bandwidth usage coefficient. Note that our model is not designed to capture bandwidth saturation for the ISPs, assuming ISPs to be smart agents with mechanisms to provide the bandwidth requested by users, and in case of zero-rating the CPs will pay for ISPs’ bandwidth.

Using Assumptions \[3\] and \[4\] and given any zero-rating strategy profile $\Theta$, we denote the utility of CP $i$ by $U_i(\theta)$ and the revenue of ISP $j$ by $R_j(\theta)$ and define them as

\[ U_j(\theta) \triangleq \sum_{j \in M} U_{ij}(\theta) \quad \text{and} \quad R_j(\theta) \triangleq \sum_{i \in N} R_{ij}(\theta), \]

where

\[ U_{ij}(\theta) \triangleq \begin{cases} q_i X_{ij}(\theta) c & \text{if } \theta_{ij} = 0, \\ (q_i - \delta_j p_j) X_{ij}(\theta) & \text{if } \theta_{ij} = 1. \end{cases} \]

and

\[ R_{ij}(\theta) \triangleq \begin{cases} p_j X_{ij}(\theta) c & \text{if } \theta_{ij} = 0, \\ \delta_j p_j X_{ij}(\theta) & \text{if } \theta_{ij} = 1. \end{cases} \]

Each CP $i$’s utility is the sum over the utilities $U_{ij}$ generated from each ISP $j$, which equals to the effective number of users $X_{ij}(\theta)$ multiplied by either its profit margin $q_i - \delta_j p_j$ if zero-rating, or its original value $q_i$ otherwise. Similarly, each ISP $j$’s revenue is the sum over the revenues $R_{ij}$ generated from each CP $i$, which equals the effective number of users $X_{ij}(\theta)$ multiplied by either the discounted price $\delta_j p_j$ if zero-rating, or by $p_j c$ otherwise. Note that the utility functions of auxiliary CPs are not well-defined since we count their users as effective users of the actual CPs they encompass, whereas the users of dummy providers do not generate any utility.

CPs’ zero-rating decisions depend on the prices imposed by ISPs, and ISPs’ decisions depend on the revenue they receive from CPs via zero-rating compared to what they would receive from users directly. Equation \[4\] and \[3\] show that CP $i$’s utility $U_i(\theta)$ depends not only on its own strategy $\theta_{ij}$, but also on all other CPs’ and ISPs’ strategies $\Theta_{-ij}$. Given the price profile $p$, ISPs make simultaneous zero-rating offers with deciding a discount profile $\delta$ to maximize their revenues, and CPs make simultaneous decisions whether or not to adopt them. We define a zero-rating equilibrium as follows:

**Definition 1 (Zero-Rating Equilibrium).** In a market of ISPs and CPs, given fixed discount and price profiles, a zero-rating strategy profile is a zero-rating equilibrium (ZRE) if and only if $1$ given a zero-rating strategy $\Theta$ chosen by ISPs, neither of CPs would gain by unilaterally deviating from $\Theta$ 2) given a zero-rating strategy $\Theta$ chosen by CPs, neither of ISPs would gain by unilaterally deviating from $\Theta$.

Based on Definition \[2\] if $\Theta$ is a ZRE, for each actual CP $i$ and ISP $j$ we have

\[ U_i(\theta_{ij}; \Theta_{-ij}) \geq U_i(\theta_{ij}; \Theta_{-ij}) \quad \text{and} \quad R_j(\theta_{ij}; \Theta_{-ij}) \geq R_j(\theta_{ij}; \Theta_{-ij}). \]

ZRE is a specific kind of Nash equilibrium \[3\], where there exist two groups of inter-dependent players. Since zero-rating is a bilateral contract between ISPs and CPs, the zero-rating decision which is affected by the entire market resembles a bargaining problem. For instance, given a pair of ISP-CP, the CP (ISP) does not have the option of zero-rating if the ISP (CP) is not willing to zero-rate. Therefore, we use the term ZRE to avoid confusion. ZRE is evaluated for a pure strategy game since mixed strategy decisions between CP $i$ and ISP $j$ to zero-rate do not apply to real-world scenarios, and users need deterministic knowledge on which ISP-CPs offer zero-rating.

For some ISP prices, although ZRE is where a CP $i$ zero-rates, it may face a utility drop compared to the case where no zero-rating is allowed in the market, i.e., when $\Theta = 0$. In this case, if CP $i$ deviates, it loses customers to the CPs who zero-rate and its utility further drops. This scenario resembles prisoner’s dilemma paradox in \[4\], where each player chooses to protect themselves at
the expense of the other participant and as a result, the optimal outcome will not be produced. However, since the market of CPs is mostly heterogeneous, this scenario mainly harms the low-value CPs rather than high-value ones as we see in Section 4.2. We define zero-rating pressure to address this phenomenon.

**Definition 2 (Zero-rating Pressure).** Zero-rating pressure happens when a CP decides to zero-rate to avoid losing customers; only because another CP is zero-rating with the same or different ISP, and if the latter cancels its zero-rating relation, the former would not zero-rate.

Suppose there are two heterogeneous CPs in the market, and in ZRE the CP with a lower value zero-rates with an ISP. In that case, its utility shall improve compared to when it does not zero-rate. If zero-rating increases low-value CP’s utility, the same must be true for the high-value CP as well. We introduce Lemma 2 to generalize this case.

**Lemma 2.** In a market \( N \) of content providers with different values, suppose \( i, i' \in N \) and \( q_i < q_{i'} \). The zero-rating strategy that CP \( i \) zero-rates with ISP \( j \) while CP \( i' \) does not is never a ZRE.

A more rigorous proof of Lemma 2 is provided in [11], which will be used in the next section to analyze how zero-rating impacts the market.

4. ANALYSIS

In this section, we represent a macroscopic and microscopic analysis of the impact of zero-rating on CPs. For the former, we evaluate the Herfindahl index [8] of the market, which is a proxy of competitiveness and looks at the CPs as a whole. For the latter, we look into individual CPs’ utilities.

4.1 Herfindahl Index Analysis

To show the impact of zero-rating on the market, we compute the Herfindahl index [8], also known as Herfindahl-Hirschman index or HHI among CPs. This index is calculated as the sum of squares over the market shares of all firms in the market, and since it accounts for the number of firms and concentration, it is an indicator of competition among firms. When this index grows to 1, the market moves from a competitive state to a monopolistic content provider, i.e., the competition decreases. Lack of competition in the market causes market distortion and significant welfare loss due to monopoly [12]. HHI increases of over 0.01 generally provoke scrutiny, although this varies from case to case [13].

In this section, we analyze a market of CPs with different values and show how the availability of zero-rating impacts the Herfindahl index of the market. The analysis of this section is based on the user model in Equation 2 and since the conclusions are theoretical, they are general to our model and are independent of parameter choices. The detailed proof of the lemmas in this section are present in the extended version [11].

**Lemma 3.** In the market \( N \) of content providers, the Herfindahl index increases when the variance of content providers’ market shares increases.

Based on Lemma 3, the more different the market shares of CPs are, the higher the Herfindahl index would be. Intuitively, a high variance between the market shares indicates that the market is moving towards a monopoly, where the increase in the Herfindahl index confirms that as well.

**Lemma 4.** The Herfindahl index is the same if none of the CPs zero-rate versus if every CP in the market zero-rates.

The amount of consumption may increase in case every CP zero-rates in the market compared to when no one zero-rates. However, since the relative market share of CPs remains unchanged (see [11] for proof), the Herfindahl index stays the same in both cases.

**Theorem 1.** In a market \( N \) of content providers with values \( \{q_1, q_2, ..., q_N\} \), suppose \( q_1 \leq q_2 \leq ... \leq q_N \), and the content providers with higher values also have higher baseline market shares, i.e., \( \delta_1 \leq \delta_2 \leq ... \leq \delta_N \). If at least one of these inequalities is strict, zero-rating options in the market will cause the Herfindahl index to be non-decreasing in all possible ZRE.

Based on Lemma 4, when zero-rating is available in the market, in case ZRE consists of either every CP or no CP zero-rate, the Herfindahl index stays the same. In other ZRE cases, if CPs with higher values and higher baseline market shares afford more zero-ratings than their low-value opponents, the Herfindahl index will increase (proof in [11]). This could represent the case where startups with low incomes and low initial baseline market shares join the market of CPs. The increase in the Herfindahl index implies that the market moves toward monopoly, where the startups would not survive.

4.2 Utility Analysis

Computing the utility for each content provider requires prior knowledge of the ZRE strategies. Note that for a two-player game (or more), neither existence nor uniqueness of Nash equilibrium could be guaranteed; it is \#P-hard to complete to determine whether the Nash equilibria with certain natural properties exist [14] and it is \#P-hard to count the Nash equilibria [15]. However, in a heterogeneous market of CPs with different values, based on Lemma 2 there are a limited number of zero-rating strategies that could become an equilibrium, i.e., the case where a low-value CP zero-rates with an ISP, while another CP with higher value does not, is never an equilibrium while the opposite can be. We focus on how zero-rating impacts CPs with different values, where incumbents and startups could be thought of as CPs with high and low values, respectively. We introduce Theorem 2 to address the special case where the low-value CP cannot afford to zero-rate in the equilibrium, while the high-value CP can.

**Theorem 2.** In a market of CPs with \( |N| \geq 2 \), let CP 1 have a lower value than CP 2. If the low-value CP does not zero-rate with any ISP \( \theta_{1,j} = 0 \forall j \in M \) while the high-value CP does \( \theta_{2,j} = 1 \forall j \in M \), and the zero-rating strategy profile for the rest of CPs (other than CP 1 and CP 2) does not change, the utility of low-value CP 1 will always decrease compared with when zero-rating is not available, while the utility of high-value CP 2 increases or remains the same.

Since utility analysis, in general, depends on the exact zero-rating strategies of the market after equilibria, which are not possible to be determined in a closed-form formula, we perform numerical evaluations to illustrate the impact of zero-rating on the CPs in next section.

5. EVALUATION AND RESULTS

In this section, we analyze the zero-rating equilibria for a market with complementary duopoly, i.e., \( |M| = |N| = 2 \), where two CPs and two ISPs compete on two sides of the market. We assume the regulation of weak content provider net neutrality [16], where each ISP charges the same price from every CP. For the simplicity of the result illustration, we also assume that ISPs’ price discount profile \( \delta = 1 \). In [11], we have separately analyzed the case where ISPs get to decide on the discount profile \( \delta \) and have shown that the results are qualitatively the same.

As Lemma 1 shows that providers with similar prices and zero-rating strategies can be merged, a duopolistic model provides a
first-order approximation of market competitions from a provider’s perspective where all its competitors are considered as an aggregated provider. We assume the elasticity of the users, baseline market shares, and prices are exogenous. Note that this evaluation can be extended to different parameter choices and is not prone to parameter selection. All prices and revenues are normalized to 1 and are not intended to reflect absolute real-world values, rather the relative differences between ISPs and CPs.

We consider two different hypothetical markets, one where zero-rating is not allowed, the other one where zero-rating is allowed, and ISPs and CPs decide on their zero-rating strategies where the market could reach the equilibria. In the extended version of this work [17], we delve into the impact of parameter selection and their impact on ZRE, HHI, and utilities, and show that parameter selection does not qualitatively change our results.

We present a benchmark scenario in which \( c = 0.5, \alpha = 0.5, \) and \( \delta = 1. \) We also have \( \phi = (0.1, 0.4, 0.4, 0.1), \) and \( \psi = (0.2, 0.4, 0.4), \) where assuming the vector indices start from 0, \( \phi_0 \) is the baseline market share of dummy CP, \( \phi_1 \) is the baseline fraction of customers who use both CP 1 and CP 2, and \( \phi_2 \) and \( \phi_3 \) are the baseline market shares of CP 1 and CP 2, respectively. Similarly, \( \psi_0 \) is the baseline market share of dummy ISP, and \( \psi_1 \) and \( \psi_2 \) are the baseline market shares of ISP 1 and ISP 2, respectively.

Figure 1 visualizes the ZRE when ISPs’ prices \( p_1 \) and \( p_2 \) vary along the x- and y-axis, respectively. Based on Lemma \( \beta \) as \( q_1 > q_1 \), 9 of the 16 possible zero-rating profiles could become ZRE under various ISP prices, which are shown in the right sub-figure as legends. The price of 0 of ISP \( j \) can represent the case where it offers an unlimited plan, and we assume in that case it is always zero-rating with all the CPs. Intuitively, when ISP prices are low, both CPs are willing to zero-rate; but when the prices are high, neither CP is willing to do so. When both CPs have values in the mid-range, they zero-rate with the effectively cheaper ISP. Furthermore, low-value CP 1 is generally more susceptible to ISP price changes; we observe that under any fixed price \( p_1 \) as \( p_2 \) increases, CP 1 first cancels its zero-rating with ISP \( j \), followed by CP 2.

Figure 2 also illustrates the regions of zero-rating pressure for CPs. The shaded blue regions demonstrate CP 1’s zero-rating pressure, and the shaded red regions demonstrate that of CP 2. When the low-value CP zero-rates with the cheaper ISP, the high-value CP as a response may zero-rate with the more expensive ISP to maintain its customers, which represents zero-rating pressure for the high-value CP. However, any zero-rating of the high-value CP can cause pressure for the low-value CP, if it is not originally willing to zero-rate.

Figure 3 visualizes the impact of zero-rating on the CPs’ utilities and HHI. We observe that in an imbalanced market of CPs, zero-rating usually decreases the utility of low-value CP 1, but increases the utility of high-value CP 2. Based on Lemma \( \beta \) (and Figure 1), high-value CP 2 always can afford more zero-ratings than low-value CP 1. Hence its utility mostly increases as it attracts more elastic users of the market. This figure also confirms Theorem \( \beta \) where in case CP 1 does not zero-rate while CP 2 does, only CP 1’s utility decreases. Figure 2(c) also shows how HHI is always non-decreasing after ZRE as opposed to when it is not allowed, which confirms Theorem \( \beta \).

6. RELATED WORK

Cheng et al. [17] and Ma [18] consider the case where CPs bargain with the monopolistic ISP to obtain exclusive priority for their traffic; CPs are charged a fee only if they opt for priority, and users can access one content provider exclusively. While they both define a fixed market share for CPs, Cheng et al. incorporate consumer surplus for a case of monopolistic ISP and find that premium peering leaves content providers worse off, but Ma assumes ISPs are always willing to offer exclusive priorities, while CPs are the decision-makers. In our work, we consider zero-rating decisions in the market of multiple CPs and ISPs and study the case where customers do not necessarily use exclusive CPs. We consider both CPs’ and ISPs’ zero-rating decisions and show that zero-rating may cause market distortion by increasing the Herfindahl index in the market of CPs, and usually leaves the low-value CP (startups with low incomes) worse off.

Reggiani et al. [19] also model an Internet broadband provider that can offer a priority to two different content providers, low-value and high-value, and show that network neutrality regulations effectively protect innovation done at the edge by small content providers. Wong et al. also consider surplus extraction by a monopolistic ISP and shows that network neutrality is necessary to ensure maximal benefit to the society. Wong et al. also formulate an analytical model of the user, CP, and ISP interactions and derives their optimal behaviors. They show that zero-rating disproportionately benefits less cost-sensitive CPs and more cost-sensitive users, exacerbating disparities among CPs. While the aforementioned models consider a market of monopolistic ISP and duopolistic CPs in which users access exclusively one content provider, our model extends to a larger market of ISPs and CPs where users are not required to access content providers exclusively.

Some previous studies focus on abolishing net neutrality under zero-rating. For instance, the authors in [22] analyze zero-rating incentives of a monopolistic ISP in a homogeneous market of consumers, and how different zero-rating equilibria impacts social welfare. Jullien et al. [23] discuss the elasticity of users and mainly focus on the case of a monopoly network with inelastic participation of consumers. While they all focus on a small monopolistic
ISP market, we extend our study to larger markets where our main focus is on how zero-rating impacts innovations in the market and we do not analyze the customers’ side in detail. Some other work study real-world markets which have established zero-rating. Mathur et al. analyze network usage data in South Africa and show that where usage-based billing is prevalent and data costs are high, users are cost-conscious where 90% of users consumed twice as much data when they do not pay for ISP bandwidth compared to when they have a usage-based mobile connection. Chen et al. also collect a dataset and by analyzing zero-rating WhatsApp on Cell-C’s network and zero-rating twitter on MTN’s network, they find that zero-rating increases overall usage of the WhatsApp on Cell-C and Twitter on MTN network while it decreases it on most other providers. While in our work we use synthetic parameters to test our model, our final results and takeaways are not qualitatively impacted by parameter choice, albeit our model is flexible to use real-world parameters as a future direction.

7. DISCUSSIONS AND CONCLUSIONS

This paper explores a controversial and unsettled aspect of net neutrality by analyzing zero-rating decisions in a market of multiple CPs and ISPs, and their impact on growing businesses and incumbents. We model the zero-rating decisions available between CPs and ISPs and find the zero-rating equilibria resulted in the market. By mainly focusing on CP’s side of the market and analyzing the Herfindahl index, we have theoretically shown the distortions in the market may increase when zero-rating is available. We further numerically and qualitatively analyze the impact of zero-rating on CPs with different values and show that zero-rating typically disadvantages low-value CPs and could stunt the innovations. Our results strongly suggest that zero-rating can be harmful to competitiveness in a market, especially when the players have asymmetric market power and hence it should be disallowed under net neutrality.

8. REFERENCES